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Application of Differentials in Movable Bridges

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Abstract

The differential has as much a wide range in mechanical designs as it does applications. Although utilization of the differential is commonplace in the modern world, the understanding of its design and application is normally limited to designers who specialize in specific machinery which utilize the mechanical devices. In moving from open gearing to manufacturer-designed reducers on movable bridges, the in-field visualization and design exposure of how these gear mechanisms function has been removed for the designer.

Two common differential designs are discussed to serve as examples. It is the purpose of this paper to provide a technical explanation of a differential's function, how it is applied, and how it can be further utilized on movable bridges.

Introduction

The differential has been used in many ways throughout history. *Confirmed* historical milestones of the differential include:

- 100 BCE–70 BCE: The Antikythera mechanism has been dated to this period. It was discovered in 1902 on a shipwreck by sponge divers, and modern research suggests that it used a differential gear to determine the angle between the ecliptic positions of the Sun and Moon, and thus the phase of the Moon.
- c. 250 CE: Chinese engineer Ma Jun creates the first well-documented south-pointing chariot, a precursor to the compass that uses differential gears to discern direction rather than a magnet.
- 1720: Joseph Williamson uses a differential gear in a clock.

The application of differentials on movable bridges has changed little since first used on swing bridges. Movable bridges currently utilize differentials almost solely for balancing torque between two outputs.

Otis Hovey[1] refers to differential gearing as an "equalizer" which describes one of its many uses. This terminology has been carried though to AREMA and AASHTO design codes today. He recommends "In cases where one motor is used to drive more than one main pinion, as in heavy swing bridges or in bascules that are driven at each truss, an effective equalizer should be introduced, preferably in the first counter shaft from the motor shaft."

Hovey appears to be a firm believer in the use of a differential and states the following warning: "In a few designs, attempts have been made to avoid equalizers and to depend on precise workmanship and installation to secure equal loads on the main pinions. When the shafts were relatively short and stiff the results were unsatisfactory, and equalizers had to be added later at much greater cost than if they had been provided in the original design."

A planetary and bevel gear differentials are presented herein for discussion of the basic function. The design principles shown can be applied to any differential design. Very often differentials are integrated with other mechanical devices, such as a clutch or mechanical speed variator, to control the speeds of differential elements. One of the more familiar examples of this would be in an automatic transmission for an automobile. In this example the clutches are electrically or hydraulically controlled to provide different gear ratios.

[1] Otis Ellis Hovey, B.S., C.E. (1927) "Movable Bridges, Volume II", New York, NY: John Wiley & Sons, Inc. Pg 4

Differential Gearing Design

Example - Epicyclic Differential

An epicyclic differential, also known as a planetary differential, consists of planet gear(s) which revolve around a center sun gear. A planet carrier, which is free to rotate, connects the centers of the gear set and keeps the planet and sun gears in mesh so that their pitch circles roll against each other. A point on the pitch circle of the planet gear traces an epicycloid curve. This gearbox design allows for a large number of gear ratios in a compact package for a very versatile range of applications.

The epicyclic differential is comprised of three principal elements, as follows:

- the case which carries the planets
- the reaction sleeve / sun gear
- the central shaft / sun gear

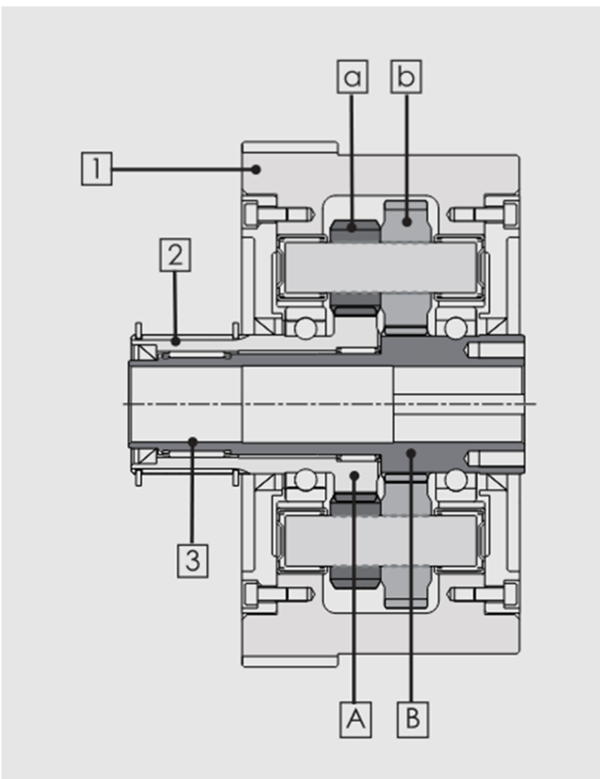

Figure 1 Shaft Mounted Epicyclic Differential Cutaway, Courtesy of Andantex Inc.

1 : case (planet carrier)
2 : reaction sleeve
3 : central shaft

A and B : number of teeth in sun gears
a and b : number of teeth in planet gears

n1 : angular speed of case
n2 : angular speed of reaction sleeve / sun gear
n3 : angular speed of central shaft / sun gear

ρ : gear ratio relating n2 to n3 (internal ratio)

T1 : torque of case
T2 : torque of reaction sleeve / sun gear
T3 : torque of central shaft / sun gear

Determine Kinematics

The relationship between the angular speeds of these differential elements is given by the Willis formula

$$n3 = \rho\ n2 + n1\ (1- \rho)$$

where $\rho = (n3 - n1) / (n2 - n1)$

OR $\rho = (A \times b) / (a \times B)$

When used as a simple reducer, the reduction ratio can be simplified to the following formula:

If $n2 = 0$
Then: $i = n1/n3$ (external ratio)
OR $i = 1 / (1- \rho)$

Note: i is positive for $\rho < 1$
$n3$ is in the same direction of rotation as $n1$

i is negative for $\rho > 1$
$n3$ is in the reverse direction of rotation as $n1$

When used as a differential, apply the Willis formula

$$n3 = \rho\, n2 + n1\,(1- \rho)$$

Determine Torque

The torque relationship of the differential elements can be determined using the law of conservation of power, summation of torques or as defined by the gear ratio. Neglecting efficiency, we have the following:

Power (in) = Power (out)
$T1 \times n1 = T2 \times n2 + T3 \times n3$

$\sum \text{Torques} = 0$
$0 = T1 + T2 + T3$

Given: $i = n1/n3$ (external ratio)
Then $i = T3 / T1$
$T3 = T1 \times i$

Given $\rho = n3 / n2$

Then $\rho = T2 / T3$
$T2 = T3 \times \rho$

It is important to note that when inertial and efficiency considerations are ignored, the torque at each of the differential elements are completely independent of the rotational speeds of these components. Since the torque relationships in the differential are fixed, output torques are set automatically based on the laws of equilibrium. The differential has one input and two outputs so that the individual output speed is not defined. Only the sum of the element speeds is defined by the Willis formula.

In many applications, gearbox heat losses should be considered in design. Mechanical losses need to be calculated to sum up bearing frictional losses, gear mesh loses, oil seal frictional losses and windage losses (oil churning). These heat losses can then be checked against the thermal capacity of the gearbox.

Example - Bevel Differential

Most commonly in the movable bridge industry, a differential is utilized having bevel satellites on the input which are in constant mesh with bevel pinions of the output shafts. The outer end faces of the bevel satellites are supported within the inner surface of a differential casing (or carrier). The kinematics and torque for this gear arrangement can be evaluated in the same way as previously provided for the epicyclic differential example.

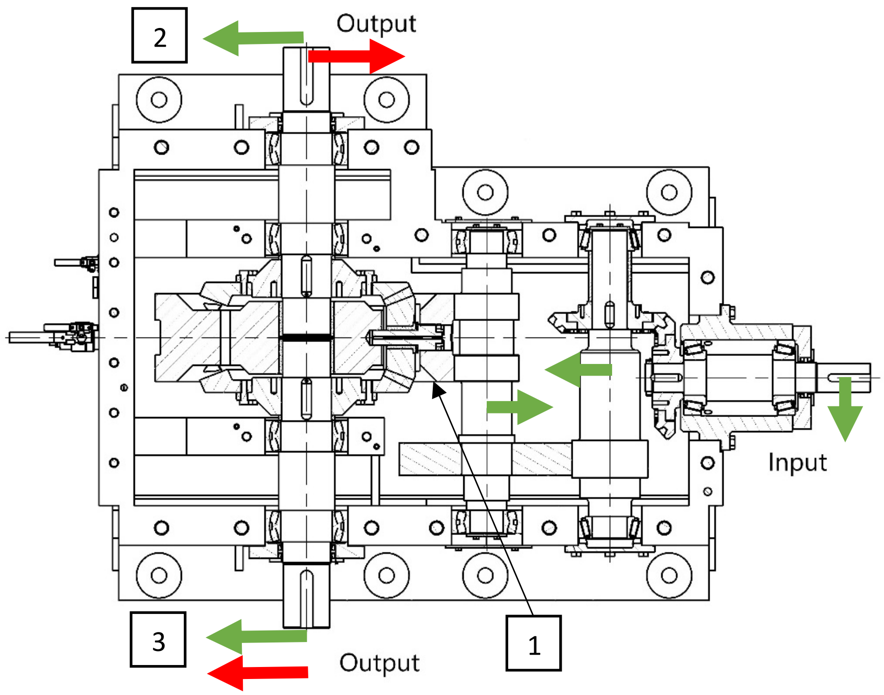

Figure 2 Bevel Gear Differential Cutaway, Courtesy of Nuttal Gear

For this application, it is the purpose of the differential to split the input torque (T1) between the two output shafts (T2 & T3). The gear ratio, which is limited and defined by the geometry, can be resolved as follows:

Holding the input shaft fixed (n1 = 0) and turning n2, n2 = -n3 (As noted by the red arrows)

Then $\rho = n2 / -n3$ (internal ratio)

$\rho = -1$

Then $i = 1 / (1 - \rho)$

$i = ½$

T3 = -T2 and T1 = 2 T3

It should be noted when output shafts turn together, n3 = n2 (As noted by the green arrows)

$n3 = \rho\ n2 + n1\ (1 - \rho)$ (Willis Formula)

$n2 = (-1)\ n2 + n1(1+1)$

n2 = n1 (All differential elements turn together, n1 = n2 = n3)

Applications in Industry

Differential gearing serves many purposes for industrial drives.

Balance Speed and Torque

In the textile industry, the use of a very long beam, necessitates a drive system which ensures uniform tension across its length. Output torque T1 = T3

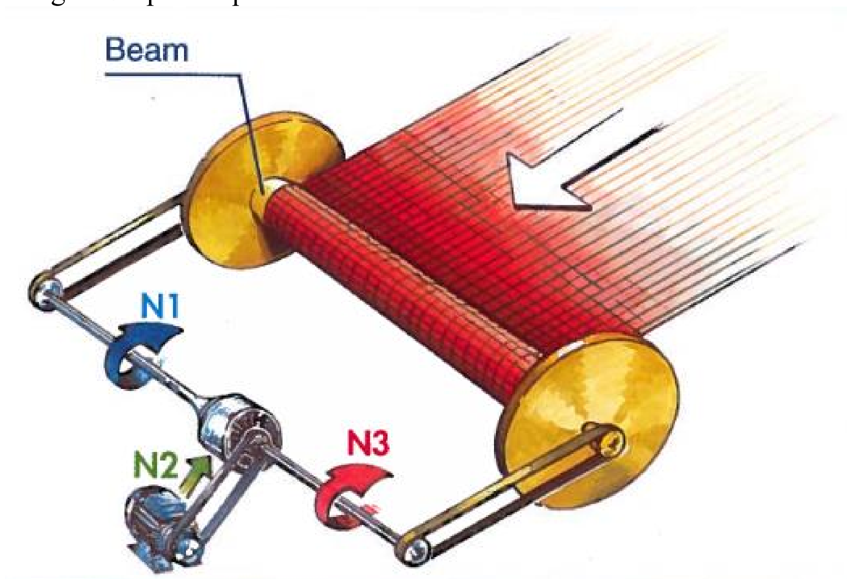

Figure 3 Courtesy of Andantex Inc.

Fragile products are often driven by a linear capstan which requires an equal level of force applied to the top and bottom belts. A differential perfectly splits drive torque and resulting forces between the two outputs.

Figure 4 Courtesy of Andantex Inc.

Precise Speed Adjustment

Sheet metal requires stretch leveling to provide flat uniform material thickness. Speed is increased slightly on the forward roller to reduce web sheet thickness.

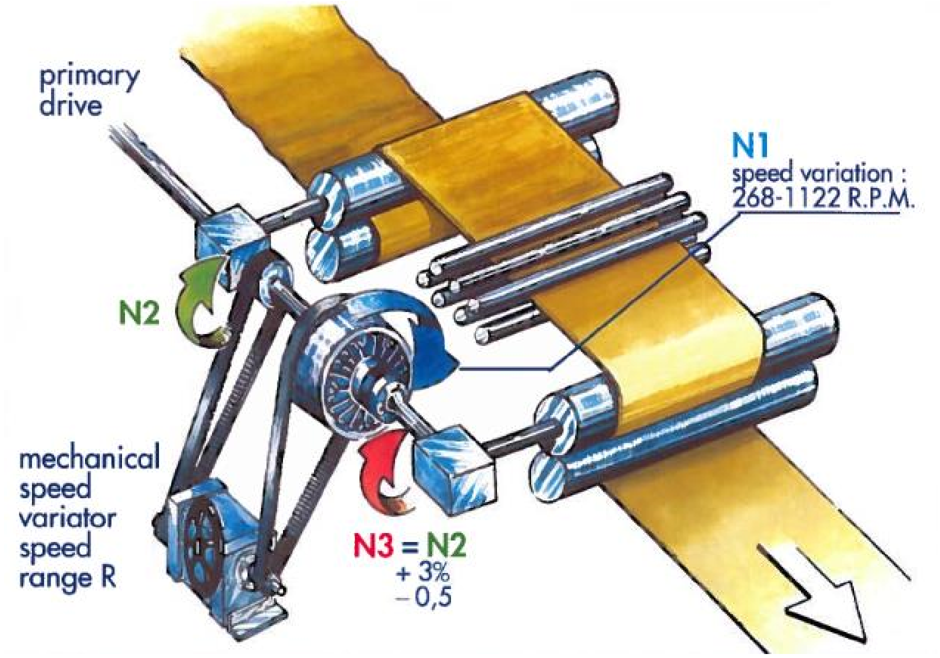

Figure 5 Courtesy of Andantex Inc.

Phase Shifting

Color printing requires multiple impression cylinders which need precise angular alignment while running. A differential provides a means to precisely adjust color registration.

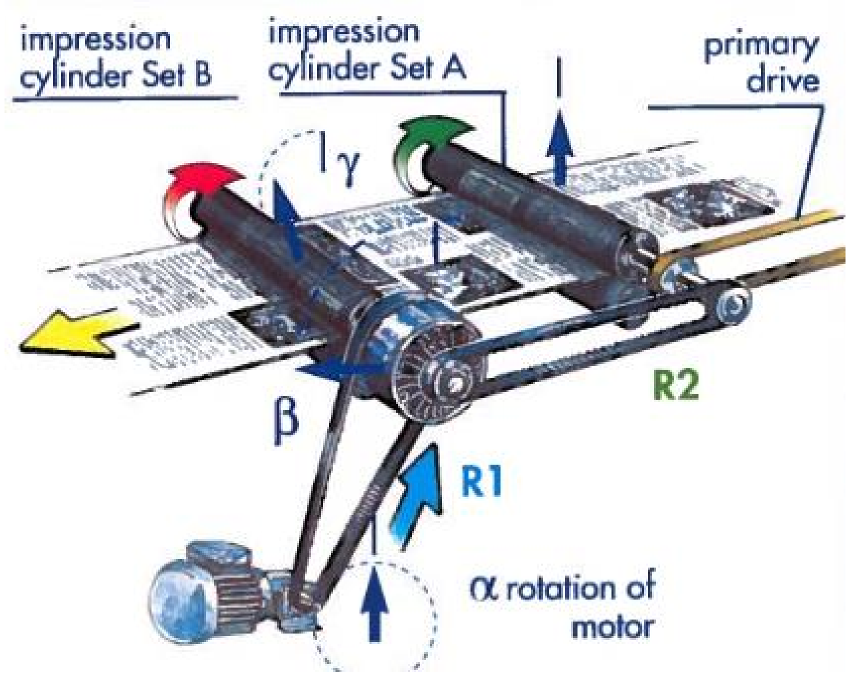

Figure 6 Courtesy of Andantex Inc.

Movable Bridge Design Code

A differential is conventionally utilized on the span drive system in two of the three basic forms of movable bridges for torque equalization between pinions. The exception is vertical lift bridges which have optimized pinion load sharing transversely by design but still requires equalization between counterweight sheaves should two be used per corner. The requirements of AASHTO "LRFD Movable Highway Bridge Design Specifications" and AREMA, Chapter 15, Part 6 "Movable Bridges" design recommendations are fairly consistent as follows:

Bascule Spans

AASHTO 6.8.1.2.1 General

Where a multiple rack and pinion drive is used, either there shall be mechanical devices, usually a differential gear reducer on the bridge to equalize the torques at the main pinions, or another such equalization method.

AREMA 6.5.35.7 Equalizing Devices

There shall be mechanical devices on bascule bridges to equalize the torques at the two main pinions, unless such equalization is provided by other means acceptable to the Engineer.

Swing Spans

AASHTO 6.8.2.1 Drive Machinery

Drive machinery for spans shall normally include drive motor(s), main reducer, output shafts, and pinions/gears driving the operating rack. There shall be a minimum of two pinions, diametrically opposite, providing equal torque to rotate the span. Either the main gear reducer shall be of the differential type, or equalization of torque shall be provided by another method acceptable to the Engineer.

AREMA 6.5.34.11 Equalizing Devices

Power operated swing spans shall have no fewer than two main pinions. These pinions shall be connected by mechanical devices which will equalize the torques at the pinions, unless such equalization is provided by other means acceptable to the Engineer.

Vertical Lift Spans

AASHTO 6.8.3.2.3 Equalizing Devices

On tower drive vertical lift spans operated through pinions engaging ring gears on the counterweight sheaves, devices shall be specified to equalize the forces at the ring gear pinions when two counterweight sheaves and two pinions are used at each corner of the span, i.e., four sheaves per tower. Equalizing devices should not be used between pinions at opposite sides of the span, but adjusting devices shall be provided between such pinions to permit leveling of the span.

AREMA 6.5.36.7 Equalizing Devices

Vertical lift spans operated through pinions engaging racks on the counterweight sheaves shall have devices to equalize the torques at the rack pinions when two counterweight sheaves and two pinions are used at each corner of the span. Equalizing devices shall not be used between pinions on opposite sides of the span, but adjusting devices shall be provided between such pinions to permit transverse leveling of each end of the span.

Vertical Lift Bridge Issues with Seating

Vertical lift bridges (VLB) have traditionally relied on both a span heavy imbalance condition, torque from the main span drive motors and span drive brakes to hold span drive torsional wind-up to ensure fully engaged seating. Span seating can be problematic on vertical lift bridges. Frequently issues arise over the life of a bridge that prevent complete contact with live load shoes in all four corners. Slipping counterweight ropes, poor imbalance, span misalignment and frictional issues are just some of the causes of these complex problems. Movable spans which are not fully seated can cause impact loading of the live shoes and adjacent structure, cause accelerated drive train fatigue / wear and can prevent span locks from engaging.

On tower driven vertical lift bridges an adjustment device between reducers and pinions which drive counterweight sheave racks permit transverse span leveling when required. Typically indexing couplings or warp adjusting clutches are disengaged by maintenance staff to allow angular adjustment, or indexing, of the shaft driving each corner counterweight sheave.

In efforts to seat a movable span, operators may impact live load shoes excessively or require the emergency attention of maintenance crews in efforts to get the span locked. The inability of operators to properly seat a span can result in extended delays for rail or vehicular traffic. Repeated attention by maintenance crews can be costly and a nuisance for owners.

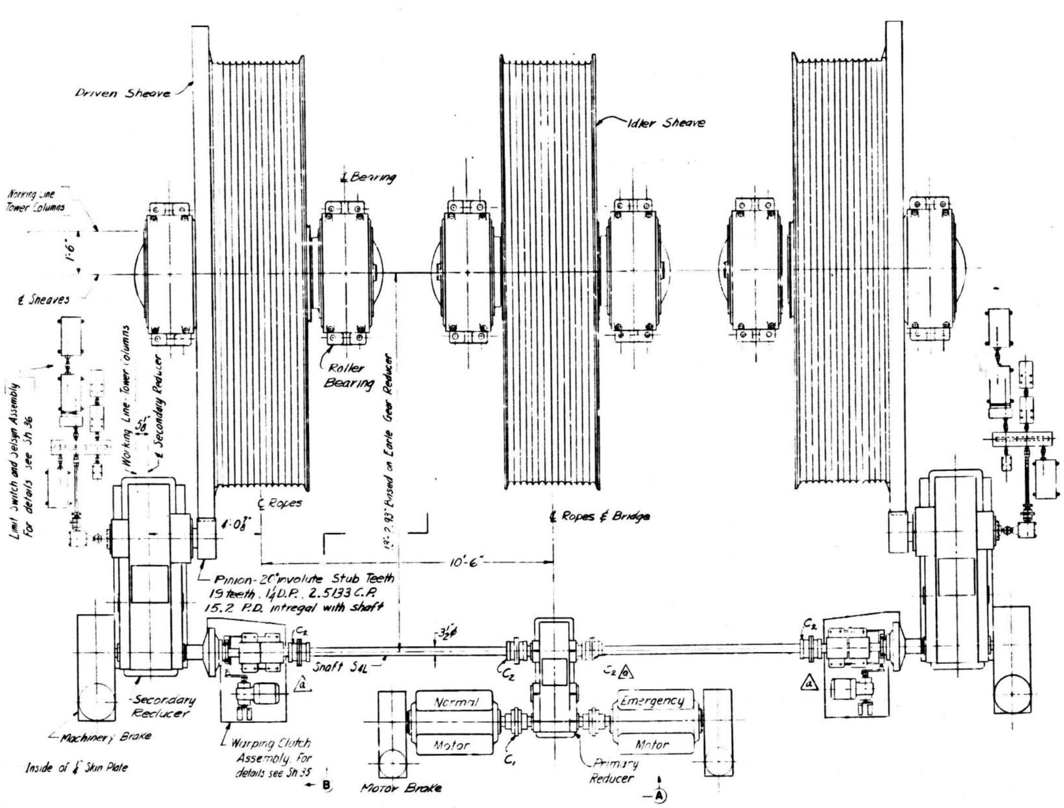

Figure 7 Plan View of Tower Vertical Lift Bridge Span Drive Machinery

Vertical Lift Bridge – Locking Differential

A unique method for span level was utilized on a historic structure that never became commonplace. The original span drive machinery utilized an open gear main reducer with a locking bevel gear differential.

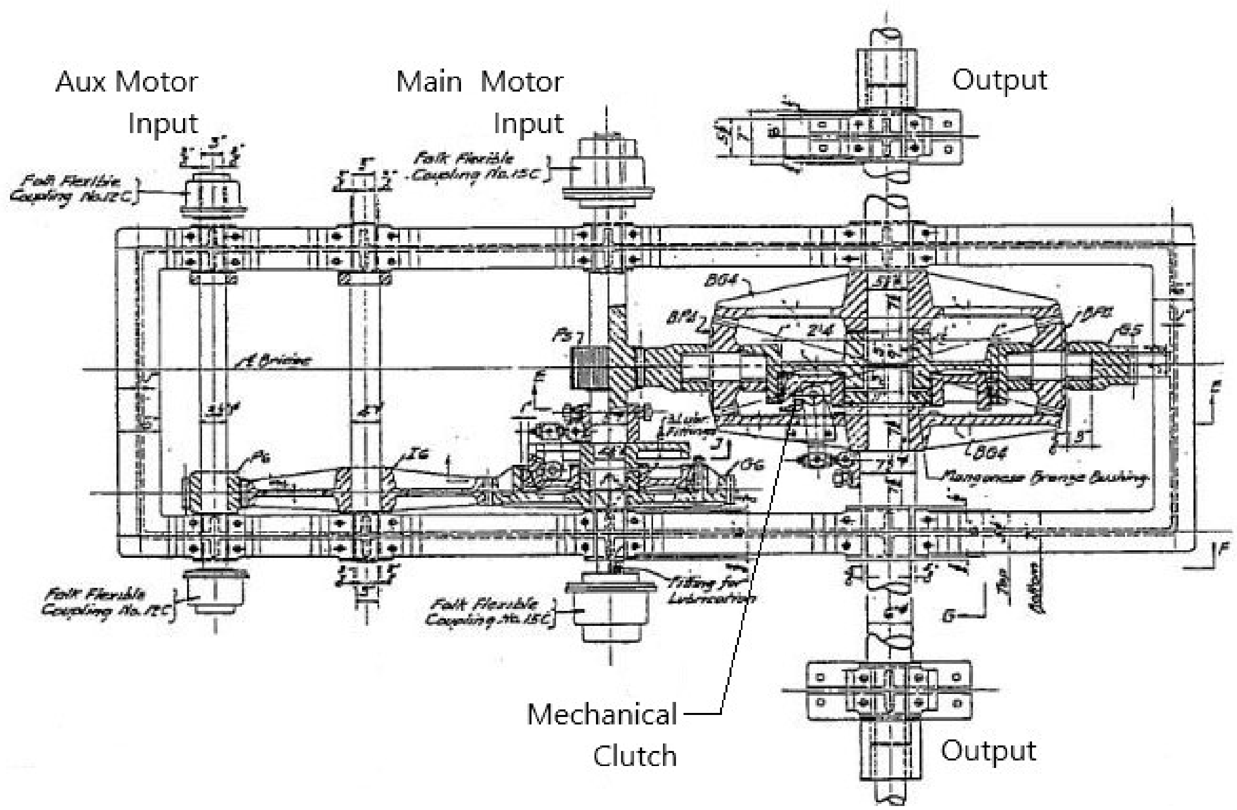

Figure 8 Main Span Drive Gearbox Cut-away Plan View, 1936 Record Drawing

During normal lift operations, a mechanical cone clutch in the differential works to lock both output shafts together. This essentially results in the reducer functioning like a standard parallel shaft reducer.

To permit leveling the span, the mechanical clutch is released which allows the differential to become active. This worked as an adjusting device between transverse counterweight pinions for span leveling since the output shafts are free to rotate in relationship to one another.

The original open gear reducer, after more than 75 years of service, was replaced in 2014 during a mechanical and electrical rehabilitation. A standard parallel shaft main reducer was installed which eliminated the differential. Other similar locking differentials have been utilized within the primary reducer on modern vertical lift bridges for span leveling.

Figure 9 Modern VLB Locking Differential

Vertical Lift Bridge Differential – Phase Shifting

One possible solution for span leveling is to utilize a differential at each counterweight pinion to adjust the angular relationship between the input and output shafts. During normal operation, power is transmitted through the planetary differential at a fixed reduction and one element remains fixed. When span leveling is required, the normally fixed element is driven utilizing a small brake motor. The gear ratio from the brake motor to the output can be designed with a high reduction to allow precise movements of the output shaft in the span's up or down direction without regard for span imbalance. When applied to all four corners of a lift span, this span leveling feature would work both in the transverse and longitudinal directions. Electronic inclinometers could be used to provide electrical feedback on the span level condition.

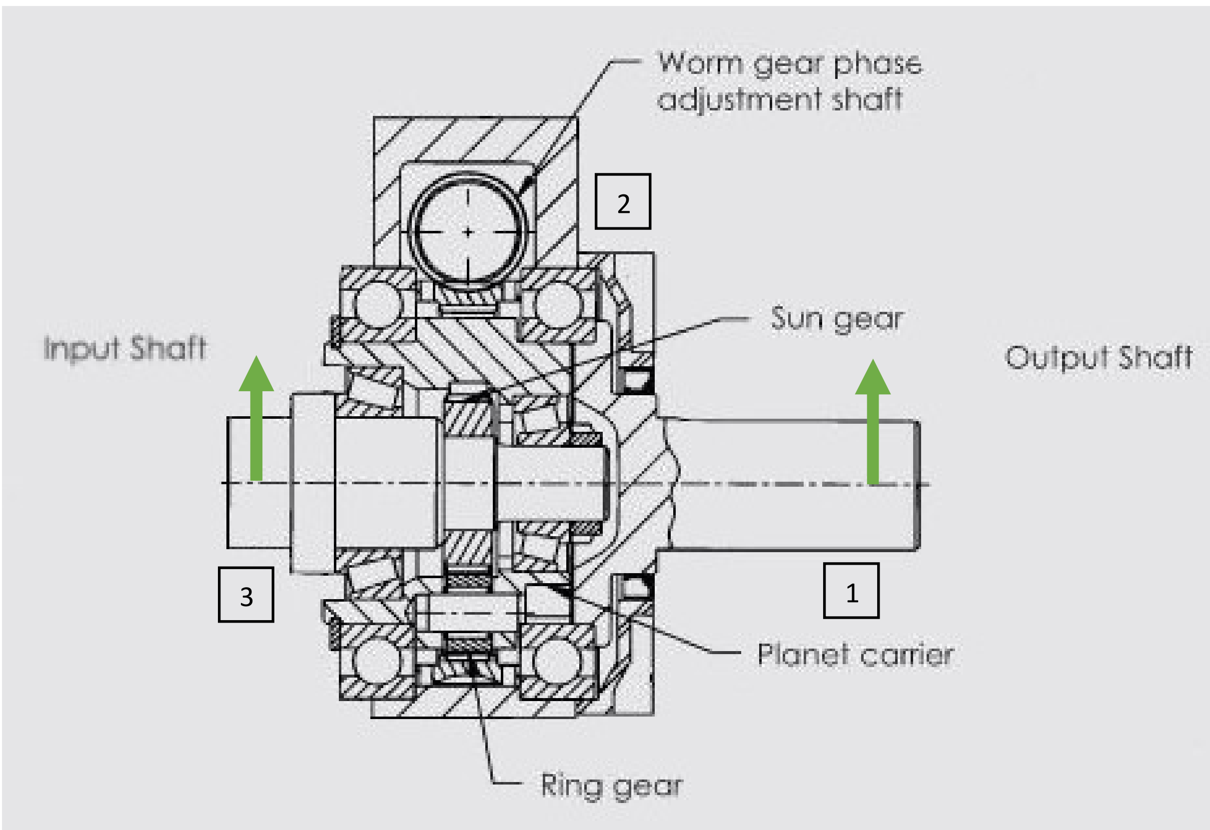

Figure 10 Phase Shifting Differential Cut-away View, Courtesy of Andantex Inc.

This design solution lends itself to remote bridge operation. New requirements for remote bridge operation require new remote solutions. Phase shifting differentials can eliminate attention by maintenance crews and provide both a time savings and remote solution to this issue. Electrically controlled phase shifting differentials would allow operators the ability to make span leveling adjustments at the push of a button, virtually eliminating related seating issues on vertical lift bridges.

The phase shifting differential, shown in Figure 10, is a commercially available gearbox design used primarily in the printing and converting market. A ring gear is driven by a worm reduction to provide an overall ratio of 135:1 from the adjustment shaft to the output shaft in a small package. The worm gearing provides a self locking, high reduction which works to prevent the adjustment shaft from being back driven.

In this example during normal operation, speed is reduced in the planetary differential 3.

$i = n1/n3$
$i = 1/3$

$i = 1/(1-\rho)$
$\rho = -2$ (internal ratio)

When phase adjustment of the output shaft takes place
Holding the input shaft fixed, $n3 = 0$

$n3 = \rho\, n2 + n1\,(1-\rho)$ (Willis Formula)
$0 = -2n2 + n1(1-{}^{-}2)$
$n2/n1 = 3/2$

Integrating a phase shifting differential into movable bridge span drive machinery would require replacing the worm gearing, as show in the example. AASHTO and AREMA generally indicates worm gearing should not be used for transmitting power to move the span. Additionally, although not required by code, specifications normally include language that through hardened gearing is required rather than case hardened gearing which is utilized commercially.

Conclusion

Otis Hovey's design recommendations, made almost a 100 years ago, have been shown to stand the test of time. A study which analyzes equalization of drive torque for pinions on a movable bridge with a differential in comparison with any other method would help to provide validity in the benefit. A second result would be to quantify factors for torque imbalance that can exist when a differential is not utilized. AASHTO design specifications and AREMA recommendations currently allow the engineer the flexibility to decide.

Movable bridges designs are diverse and are in continuous development. Mechanical engineering challenges are presented on every project, whether it is new work that features the latest technology or rehabilitation of a 100 year old structure.

Differentials are a versatile mechanical device which can be utilized in innovative ways. A designer's understanding of a differentials function and application possibilities can provide solutions to movable bridge drive problems using gearing which has been proven to be both reliable and to provide a long service life.