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Thinking Outside the Box – Using Small Diameter Sheaves

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Introduction

AASHTO Standard Specifications for Movable Highway Bridges has provisions for the use of small diameter sheaves. The provisions are seldom used and may not be understood by most.

This paper will discuss the requirements of the AASHTO Movable Bridge Specifications and how small diameter sheaves can be used in movable bridge applications. This paper will also discuss rope fatigue in general in order to provide some back up as to how and why these provisions were established.

This paper will show use the AASHTO provisions for small diameter sheaves to prepare alternate designs, and the cost of various alternatives will be compared. A preliminary design for the Illinois Street Intermodal Bridge will be presented as an example.

Sheaves- What are they?

Sheaves are circular assemblies either castings or weldments used to support or change the direction of a moving wire rope.

The following are some typical wire rope applications that use sheaves to either support or change the direction of wire rope:

- Change the direction of main or auxiliary counterweight ropes on vertical lift bridges
- Support or change the direction of operating ropes on a span drive vertical lift bridge
- Support or change the direction of equalizing ropes on a towerless vertical lift bridge.
- Support or change the direction of ski lift ropes
- Cranes
- Elevators
- Etc.

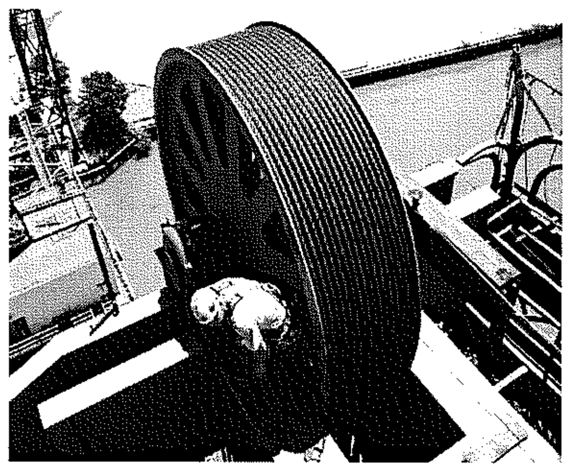


Photo 1: Counterweight sheaves at West 3rd Street in Cleveland, Ohio

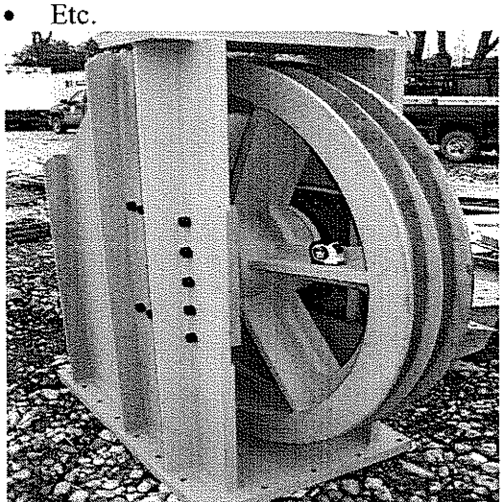


Photo 2: Deflector sheaves at West 3rd Street in Cleveland, Ohio

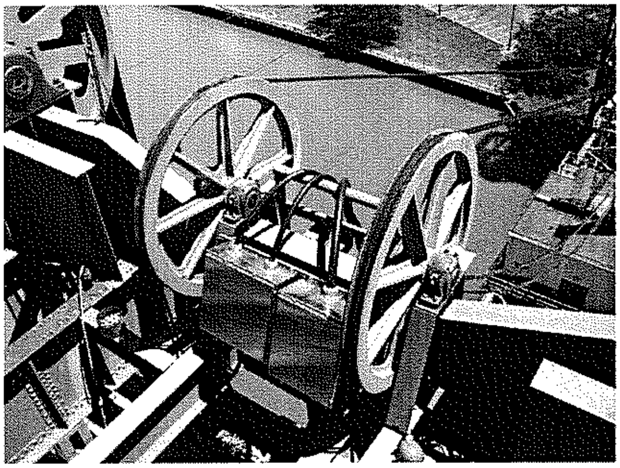


Photo 3: Auxiliary counterweight sheaves and messenger cables at West 3rd Street in Cleveland, Ohio

Sheaves – How do they work?

Sheaves are designed to transmit loading from the wire ropes to the shafts and bearings that support the sheaves. The magnitude of the loading is dependent on the load in the wire ropes and the change in rope angle. In the case of a counterweight rope (see figure 1) the rope typically changes direction 180 degrees. Therefore the load on the sheave is equal to two times the load in the wire ropes. This load is distributed along half of the diameter of the sheave. In addition the rope fits in a rope grove distributing the load over half of the rope diameter. In the case of deflector sheaves the rope is bent over a smaller angle (see figure 2)

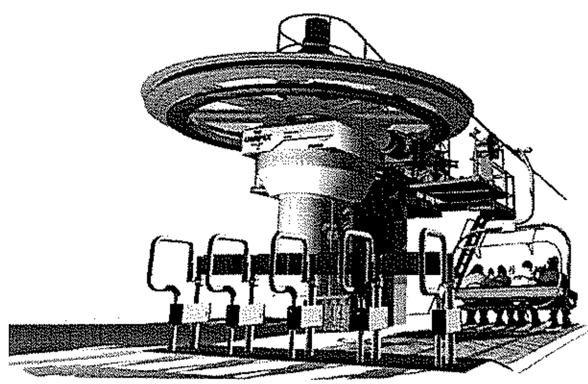
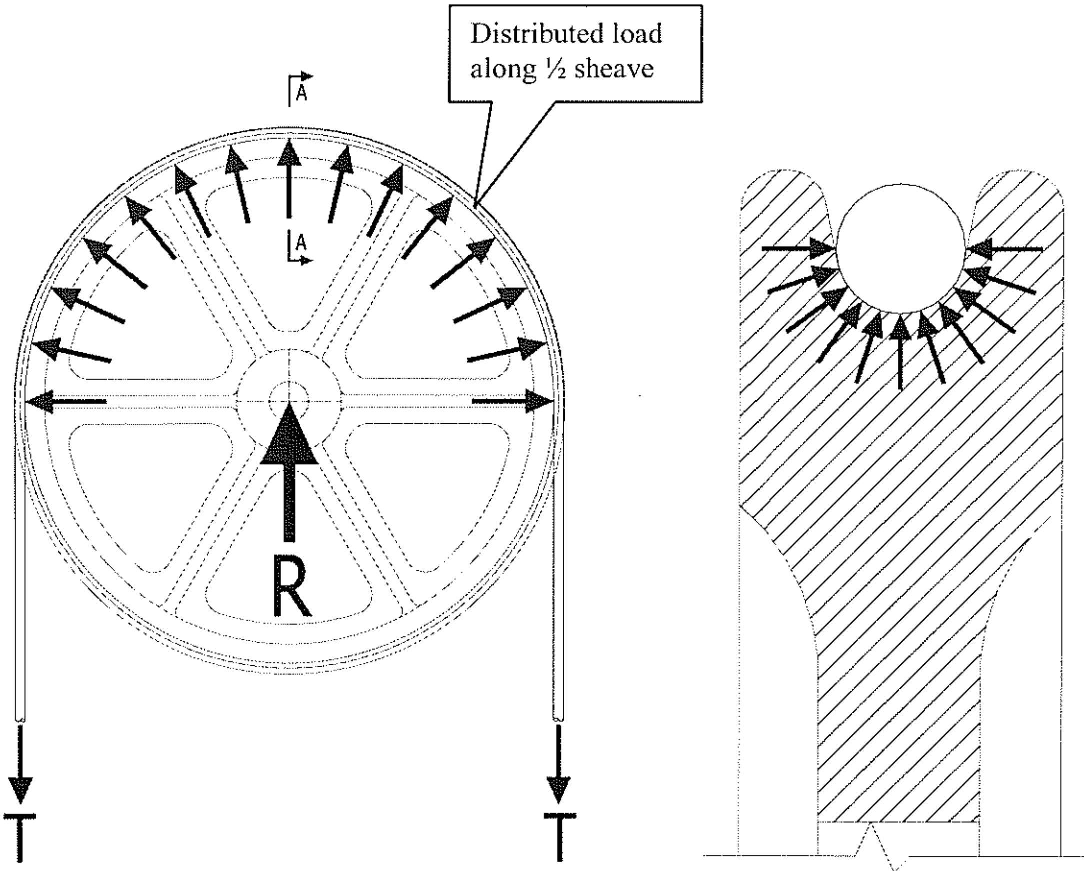


Photo 4: Large Sheave Applications for Ski Lifts,

Sheaves must also be sized to minimize bending and fatigue in the wire rope, and therefore increase the rope life. As per literature available from Haines Supply, Inc., a supplier of wire rope, "If ropes operate over inadequately sized sheaves the severe bending stresses imposed will cause wires to break from fatigue even though actual wear is slight. The smaller the diameter of the sheave, the sooner these fatigue breaks will occur and the shorter rope life becomes."

As you can see sheave sizes affect the stresses in the sheaves as well as the stresses in the wire ropes, but what are the requirements for sheave size and what the impacts of the sheave selection are. These are the topics that this paper will cover.



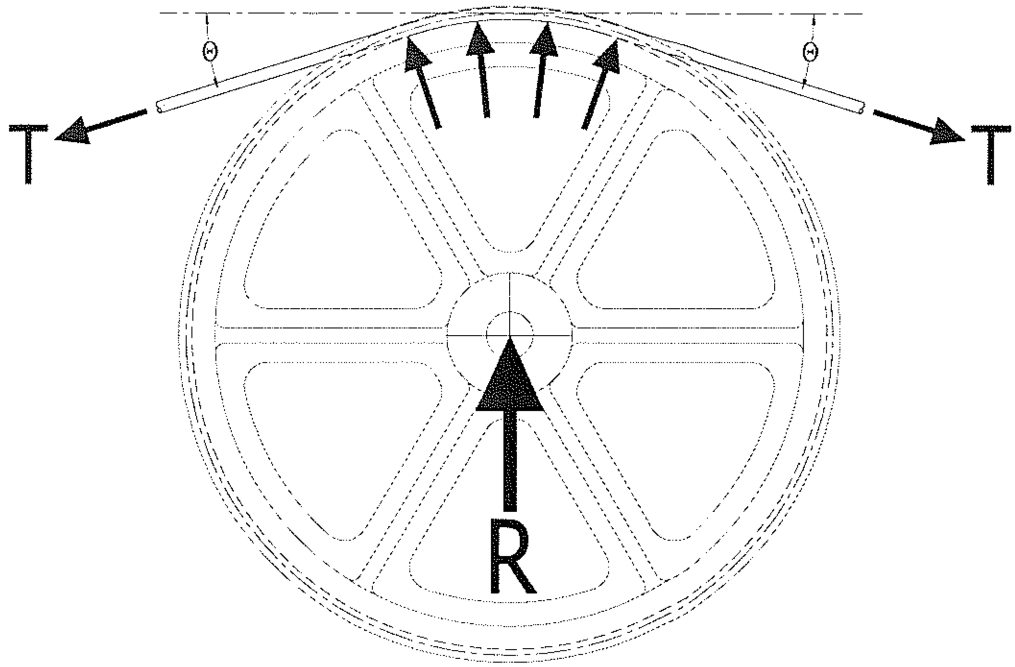
R = 2T and P = 2T/Dd

<u>Section A-A</u>

Where,

P : Pressure (psi)
T : Tension (pounds)
D : Sheave Diameter (in)
D : Rope Diameter (in)

Figure 1



$R = 2T \sin(\theta)$ and $P = 2T/Dd$

Figure 2

Design Requirements

Many factors come into play in determining sheave size including industry standards, wire rope construction, application, and the stress and fatigue in the wire ropes and the sheaves.

Every industry that uses wire rope for haulage or lifting has its own set of standards. For example a table of proper sheave and drum sizes is available on the Hanes Supply web site. Hanes Supply is a firm specializing in lifting solutions and wire rope splicing since 1930. The sheave and drum size table indicates the minimum and suggested diameters as a ratio of the tread diameter to the nominal rope diameter or "D/d ratio". The D/d ratios presented vary significantly with rope construction. The more flexible wire ropes can be bent at tighter radius with out adversely impacting the wire rope. The table reflects the wide range of wire rope that might be used for lifting operations during construction. The proper sheave and drum diameters as presented on the Haines Supply web site are:

Construction	Suggested D/d * Ratio	Minimum D/d * Ratio
6 x 7	72	42
19 x 17 or 18 x17 Rotation Resistant	51	34
6 x 19 Seale	51	34
6 x 27 H flattened strand	45	30
6 x 31 V flattened strand	45	30
6 x 21 filler wire	45	30
6 x 25 filler wire	39	26
6 x 31 Warrington Seale	39	26
6 x 36 Warrington Seale	35	23
8 x 19 Seale	41	27
8 x 25 filler wire	32	21
6 x 41 Warrington Seale	32	21
6 x 42 filler	21	14

*D = tread diameter or sheave,
d = Nominal diameter of rope

Wire Rope Construction

The table presented above shows a wide range for proper sheave sizes. The large range in sheave sizes corresponds to the wide range of wire ropes and their mechanical properties. To understand this better it is important to understand wire rope construction and the nature of wire rope fatigue. Wire rope is made up of a number of stands laid helically about a metallic or non- metallic core. Each strand consists of a number of wires also laid about a metallic or non-metallic center. There are different types of wire rope construction as noted for different conditions and usage. The wire rope construction varies with the type of core, the number of strands, the number, size, material and arrangement of the wires within the strands and how the strands are laid or woven together.

The following information is taken from the Machinery Handbook 25:

- **Rope Wire Materials:** Materials used in the manufacture of wire rope are, in increasing order of strength: iron. Phosphor Bronze, traction steel, plow steel, improved plow steel, and Bridge rope steel.
- **Rope cores**: Wire rope cores are made of fiber, cotton, asbestos, polyvinyl plastic, a small wire rope (Independent Wire Rope Core), a multiple wire strand (wire-strand core), and cold drawn wire-wound spring.
- **Wire-Rope Lay**: The lay of a wire rope is the direction of the helical path in which the strands are laid. If the wires in the strand or strands in the rope form a helix similar to threads of a right hand screw, i.e., they are would around to the right they are called right hand and conversely, if they wind around to the left the lay is called left lay. Regular lay means that the wires in the strand are laid opposite to the lay of the strands in the rope. For example a right regular lay has the strand laid to the right and wires within the strands are laid to the left.
- **Strand Construction:** Various arrangements of wire are used in the construction of wire rope strands these are: Standard Coarse Laid, "filler-wire", Warrington, Seale, and flattened

strand. The standard coarse laid strand has 6 wires wrapped around a central wire of the same size. Table showing weights and strengths of various wire rope constructions are shown in the appendix.

Properties of Wire Rope

As per the Machinery's Handbook 25: "Important properties of wire rope are strength are strength, wear resistance, flexibility and resistance to crushing and distortion".

Strength: The Strength of wire rope depends upon it size, kind of material of which the wires are made and their number, the type of core and whether the wire is galvanized or not. .....

Wear Resistance: Wear Resistance depends on the size and material of the outer wires, the wire rope lay (Lang lay were wires and strand are wrapped in the same direction reduces wear), and the strand construction (flattened strand exhibit less wear).

Flexibility: Flexibility is dependent on size of the wires (a greater number of smaller size wires provides greater flexibility), the wire rope lay (Lang Lay provides greater flexibility), and rope core (Wire rope with independent wire rope core performs better in bending since the core provides better support to the outer wires).

Standard Classes of Wire Rope: Wire rope is typically classified with two numbers. The first number is the number of strands and the second number is the number of wires in a strand. For example a 6 x 7 class has six strands with seven wires per strand. The most common hoisting ropes are the 6 x 19 class which have six strands with nineteen wires per strand. Some hoisting ropes in this class have filler wires in the construction of the strands. A 6 x 25 filler wire rope has 6 strands with 9 wires and 6 filler wires per strand or 25 wires total per strand. These are the types of ropes are used in movable bridge construction.

AASHTO 3.2.3 requires that ropes be made of improved plow steel wire. All operating rope shall be preformed wire rope.

All ropes to be 6 x 19 Class wire rope of 6 x 25 filler wire construction. Each strand shall consist of 19 main wires and 6 filler wires fabricated in one operation, with all wires interlocking.
There shall be four sizes of wire in each strand: 12 outer wires in one size, 6 filler wires of one size, 6 inner wires of one size, and a core wire. Jute core shall not be used.

Wire Rope Fatigue

All ropes that are bent over sheaves or drums are subjected to bending stresses and therefore are subject to fatigue do to these stresses. The fatigue life of the rope is dependent on the degree of rope bending, load in the rope, rope construction, number of cycles, rope speed, rope lubrication and the condition of the sheaves or drums. While all of these factors affect the fatigue life of the rope we will focus mainly on the fatigue due to bending.

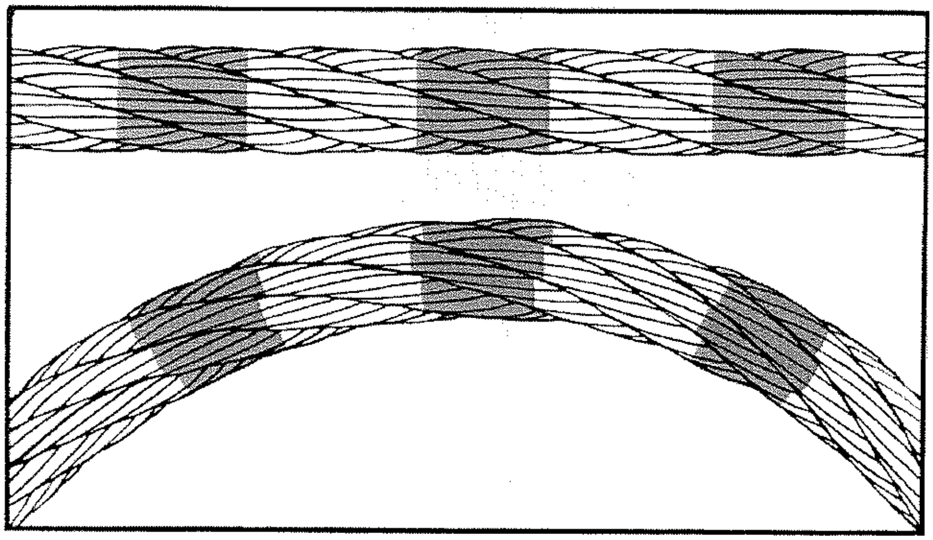


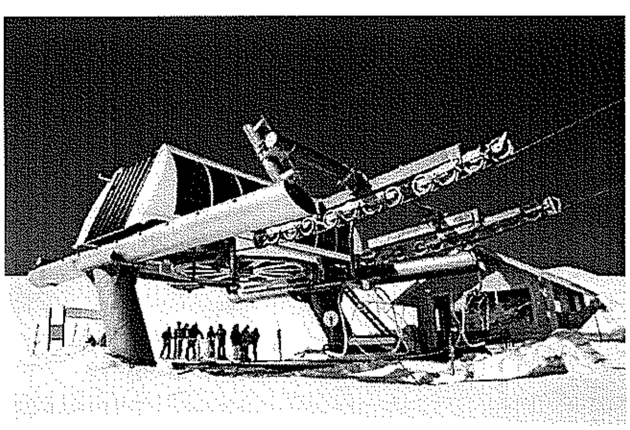
Figure 3

The amount of bending stress in the wire rope will depend on the ratio of the sheave or drum diameter to the rope diameter and the tension in the rope. As a wire rope is bent over a sheave the strands and wires move relative to each other to account for the differences between the inside and outside diameters of the wire rope. Obviously the bend radius and arc lengths are smaller directly against the sheave than along the outside edge of the rope. Due to the helical construction of wire rope the differences in strand lengths are not so obvious. Figure 3 shows a view of wire rope before and after bending. Note the relative movement of the strands by observing the shaded sections before and after bending. Excessive bending results in excessive movement between the strands and greater fatigue to the wire rope.

Rope Bending

Full bending of wire rope occurs when a length of wire rope in contact with a sheave is equal to one rope lay. Therefore for if a 1 inch diameter wire rope with a 6 inch rope lay is bent over a 54 inch diameter sheave, full bending would happen in 12°-44'. This is seen by multiplying 360°by the ratio of the rope lay to the diameter of the sheave or $360° \times 6" / 54\pi"$

This bending happens at the point in which the rope first comes into contact with the sheave, and it doesn't matter the length of arc through which the rope is turned in excess of 12°-44'. So bending the wire rope 30°, 60° or 90° has the same effect on the wire rope. This can be understood by realizing that the strands are forced to move relative to each other in the first rope lay of contact with the sheave. It is here that the rope must change from a straight rope to a bent rope. It doesn't matter how far the rope travels as long as it maintains the same curvature.



Conversely bending a 1 inch diameter rope through an angle smaller than 12°-44' would result in less bending and fatigue to the wire rope. This is because the strands would not be forced to shift as much with respect to each other. Therefore when less than full bending occurs smaller sheaves could be used without damaging the ropes. Thus a small diameter sheave when less than full

oto 4: Small diameter sheaves for ski lifts

bending is encountered is equivalent to a larger diameter sheave with full bending, and the smaller the angle of departure the smaller the required sheave size. Small diameter sheaves are commonly used for small angles of departure particularly where space constraints would make a single large sheave impractical. Small diameter sheaves very common in the ski industry as noted in Photo 4. An off the shelf application of small sheave or rollers can be seen in Photo 5. In fact the AASHTO Standard Specifications for Movable Highway Bridges has provisions for small diameter sheaves which we will discuss further.

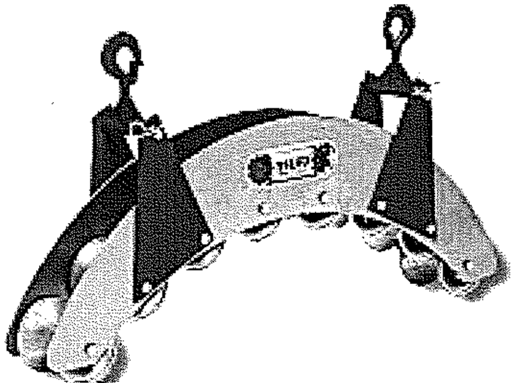


Photo 5 : Prepackaged Small Diameter Sheaves

There are issues other then bending that most be considered when sizing sheaves. It was previously noted in figures 1 and 2 that the rope groove pressure is inversely proportional to the diameter of the sheave regardless of the angle of departure. This means that if a small diameter sheave is used to turn wire rope through a small angle the groove pressure is increased. Therefore the groove pressure must be considered in the sheave selection.

AASHTO Requirements

AASHTO criteria for sheave diameter vary with application. Counterweight ropes, auxiliary counterweight ropes and operating ropes have different recommended sheave diameters as can be seen by the following subsections of the AASHTO Standard Specifications for Movable Highway Bridges:

2.9.7. Counterweight Sheaves

For the main counterweight ropes, the pitch diameter of sheaves, center to center of ropes, shall not be less than 72 times the diameter of the rope, and preferably not less than 80 times the diameter of the rope. For auxiliary counterweight ropes, the pitch diameter for the sheave shall not be less than 60 times the rope diameter.

....

2.9.8. Operating Drums and Deflector Sheaves

For operating ropes, the diameter of the drums and deflector sheaves shall not be less than 45 times the diameter of the rope, and preferably not less than 48 times, except for deflector sheaves with small angles of contact between rope and sheave.

Operating drums shall have pressed fits on their shafts, and in addition shall have keys designed to carry the total torque to be transmitted by the shafts.

The shape of the groove on operating drums shall conform as closely as feasible to the rope section. The distance center to center of rope grooves shall not be less than 1/8 inch more than the diameter of the rope.

Deflector sheaves shall generally have the same diameter as the drums. Intermediate deflector sheaves shall be provided as necessary to prevent rubbing of the ropes on other parts or to avoid excessive sag of the ropes. When operating ropes have small angles of contact with deflector sheaves the sheaves shall be supported on roller or ball bearings and shall be designed as light as practicable to insure easy turning and minimum rope slippage in starting and stopping.

All deflector sheaves shall have deep grooves to prevent displacement of the ropes.

These two subsections establish minimum D/d ratios for counterweight and deflector sheaves. Traditional, the number and size of ropes can be established based on the total load. The recommended D/d ratio based on the application can be used to establish the sheave diameter. The stresses in the rope and sheaves calculated and the size of the sheaves would be adjusted as required. In addition to the direct tension load the bending stress in the wire rope must be calculated based on the diameter of the sheave. AASHTO section 2.5.20 Bending stress and Allowable load over Sheave provides a formula for computing the bending stress in the wire rope. Please note that formulas for computing bending stresses in wire rope vary significantly. Table B in the appendix provides 8 different formulas for computing bending stresses in wire rope. The table comes from material provided at a wire rope seminar by Wally Stack. Please note that the range of bending stresses calculated by these formulas vary by more than 90%.

As per Article **2.5.20 Bending Stress and Allowable Load over Sheave**:

If wire rope is bent over a sheave, the bending stress and allowable load on the rope shall be calculated as follows:

Let P = allowable load on rope, in pounds
K = unit stress due to bending in extreme fiber of largest individual wire in psi
E = Modulus of Elasticity 28,500,000 psi
a = cross sectional area
d = diameter of outer wire in inches
D = diameter of sheave, center to center of rope in inches
S = greatest unit tension allowable
L = angle of helical wire with axis of strand
B = angle of helical strand with axis of rope
C = diameter of rope

Then

$K = 0.8\ Ed/D \cos^2 L \cos^2 B$ (1)

P= a (s- .8ed/D cos2L cos2 B) = a (s- 0.7 ed/D) (2)

For wire rope having 6 strands of 19 main wire each (6x25 filler wire construction and assuming c/16.

P= a (s- 1,300,000c/D) = a (s- 8963.2c/D) (3)

For haulage rope, 6 strands of 7 wires each, d=c/9

Value of P shall not exceed the values in Art. 2.5.19

Now for a small diameter sheave Article 2.5.21 **Small Sheave over Short Arc** states:

If a rope is in contact with a small sheave over a short arc (50° or less), the actual radius of curvature of the rope may be grater than that of the sheave.

Let R = the actual radius of curvature of the rope
θ = the angle between the directions of the ropes
W = pull of the individual wire (equals P divided by number of wires if all wires are of equal diameters)

Then

$R = d^2/(4.25 \sin(\theta/2) \sqrt{E/W}$

If R is greater than the radius of curvature of the sheave 2R should be used in place of D in the formulas (1), (2), (3) Art. 2.5.20.

Small Sheave Uses

Now what does Article 2.5.21 mean and where would these small sheaves be used? Article 2.5.21 tells use that when bending a wire rope through a small angle over a small sheave the radius of curvature of the rope is actually larger than the diameter of the sheave. The article gives us a formula to compute the radius of curvature of the rope and we can use this radius of curvature in computing the bending stresses. You also need to check rope groove pressures, shear, bending, and axial stresses in the sheaves.

The typical application for small sheaves is for intermediate deflectors. When the operating ropes pass over the intermediate deflector sheaves the rope sees a very small angle change. The main purpose is to take out the sag in the rope because the operating ropes typically have long horizontal runs. In these instances where the angle of departure is so small a sheave could be used that has a diameter much smaller than minimum of 45 times the rope diameter as specified in Article 2.9.8. The smaller diameter sheave reduces weight on the lift span. It also reduces the moment of inertia of the sheave which makes the sheave easier to turn thereby reducing wear on the rope due to rope slippage over the sheave. The ASSHTO LRFD Movable Highway Bridge Design Specification has changes the requirements for small sheaves. Article 6.8.3.3.5 now says: When operating ropes have less than a 45 degree arc of contact with a deflector sheave, the minimum sheave diameter shall be at least 26 times the wire rope diameter. The commentary provides the explanation that when a rope is in contact with a small deflector sheave over a short arc taken as 45 degrees or less the actual radius of curvature is usually larger than the deflector sheave radius. Unfortunately the formula for calculating the radius of curvature has been eliminated. It is also unfortunate the wording was changed to tie the use small sheaves to deflectors.

The important issue for the wire rope fatigue is: when you turn a wire rope through a small angle the effective radius of curvature is larger than the radius of the sheave. The effective radius of curvature is used in determining rope bending stresses and therefore fatigue life. No this is true for all rope regardless of the application. The bending stress calculations are the same for operating ropes or counterweight ropes, and the effect of small sheaves is also the same regardless of the application. Let's look at a large sheave that turns a rope through an angle of 180° (see figure 3).

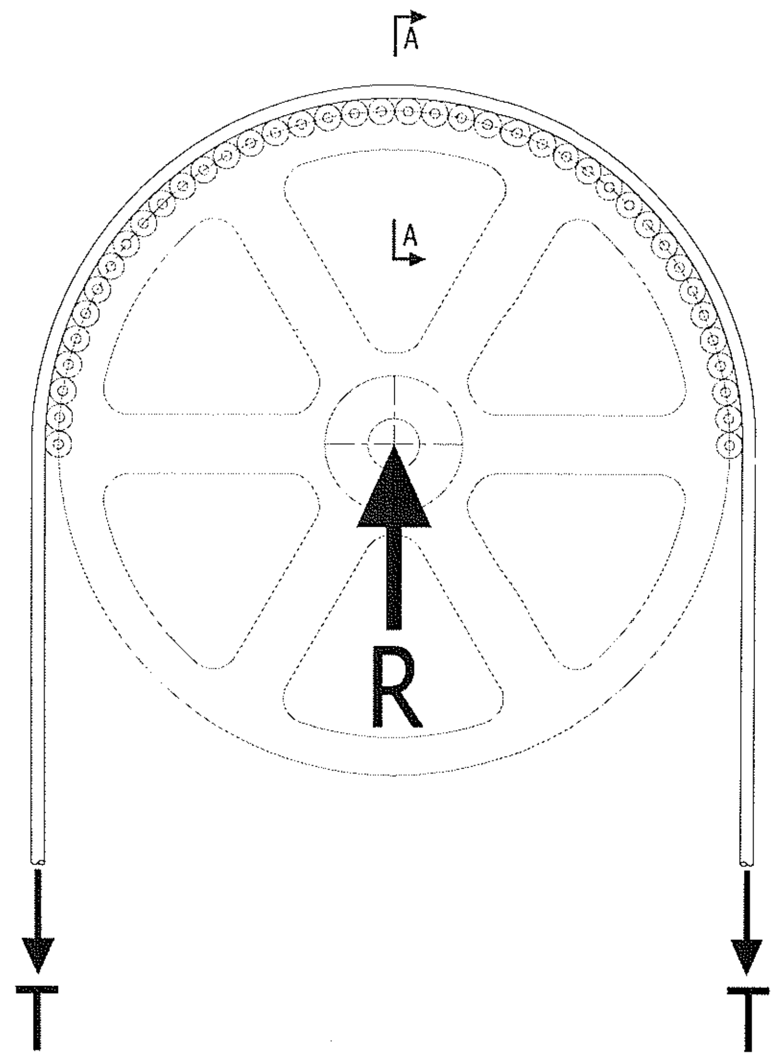


Figure 4

If the large diameter sheave were replaced with a series of small diameter sheaves what affect would this have on the wire rope? If there were an infinite number of infinitely small sheaves it is obvious that the results would be the same as one large sheave. In other words the effective radius of curvature would be the same as the large sheave diameter. If the effective radius of the wire rope were 72 times the rope diameter than this condition would satisfy the AASHTO criteria for a counterweight sheave. Now obviously constructing an infinite number of infinitely small sheaves is infinitely less than possible. By contrast Figure 4 shows the same large sheave replaced with two very small sheaves, and each sheave has a 90° angle of departure. It should also be obvious by observation that bending the wire rope through a 90° angle over a small radius would create a severe bending condition that would significantly reduce the fatigue life of the wire rope. So what are the limitations on using small sheaves, and what are the benefits of replacing a large sheave with a series of smaller sheaves?

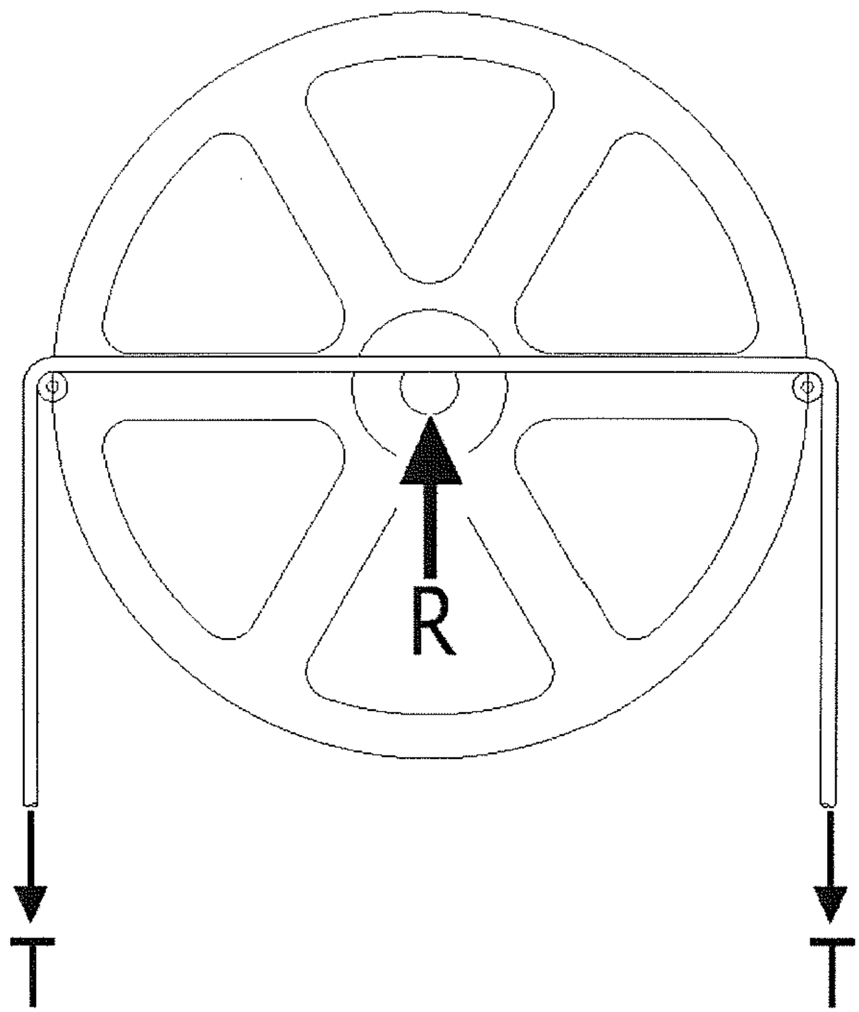


Figure 5

Small Sheave Design

Article 2.5.21 of the AASHTO Standard Specifications for Movable Highway Bridges provides a methodology to design a series of small sheaves that could be used to replace a single large sheave. A small sheave design should try to achieve a compromise between the extremes presented in figures 3 and 4. As such, we have analyzed wire rope bent through 180 degree angle using various numbers of sheaves, sheave sizes and angles of departure. We will determine if they comply with the requirements presented in the AASHTO Standard Specifications for Movable Highway Bridges, and we will develop a construction cost for each as a method of comparison.

For comparison purposes we will use a 1 inch diameter 6 x 19 wire rope of 6 x 25 filler wire construction with 5,000 lbs of total unit tension applied. For the purpose of discussion we will assume that this is an operating rope and as such the total unit tension must be less than 3/10ths of the ultimate strength of the rope, or 60,900 psi, as per the requirements of Article 2.5.19. We will start with a single large sheave, and then compare that to various alternative configurations. The configurations selected for analysis were obtained by fitting an increasing number of sheaves into the outline of the initial large sheave. In other words the center to center dimension for the wire rope over a 180° bend will be the same for each case with in a given group. We have looked at a number of initial single sheaves that range from 54 inches in diameter to 84 inches in diameter. The smaller diameter sheaves would meet the criteria for operating ropes the larger diameter sheaves would meet the criteria for counterweight sheaves.

For each case we have we have gone through a series of design steps as follows:

1. We calculate the angle of departure or bending angle that the rope sees over each sheave. This angle is compared to the definition of a small sheave as presented in Article 2.5.21 of the AASHTO Standard Specifications for Movable Highway Bridges. If the angle is greater than 50° no further analysis is performed.
2. We calculate the arc length that the rope moves over the sheave. AASHTO has no limitation on arc length. We believe based on information presented elsewhere in the paper that the arc length should be less than one rope lay. The reason being that full bending occurs in the first arch length. Once full bending is encountered the angle no longer matters. In other words the effective radius would be the same as the sheave radius. Therefore if the arc length is greater than one rope lay the configuration is rejected and no further analysis is provided.
3. The actual radius of the rope is calculated as per the equation in Article 2.5.21.
4. The actual radius calculated is compared to the radius of the small sheave. If the calculated radius is larger than the small sheave radius than the larger radius is used in calculating the design diameter. While not specified in AAHTO we have placed a limit on the design sheave diameter. By observation the design diameter can never be greater than the centerline to centerline of rope. In other words the design diameter using small sheaves to replace one larger sheave can never be larger than the initial sheave diameter.
5. The bending stress is calculated, in accordance with Article 2.5.20 of the AASHTO Standard Specifications for Movable Highway Bridges. The bending stress is subtracted from the greatest unit tension allowed. The difference is multiplied by the cross section of the rope to yield the allowable load on the rope. The allowable load on the rope is then compared to the 5,000 pound applied load. As such we have determined if the rope bending meets the AASHTO requirements for Small Sheaves over Short Arcs.
6. Additionally we have calculated the groove pressure for the smaller sheave diameter and compared this to an allowable bearing pressure of 13,000 psi on sheave, assuming grade 36 cast steel construction.

7. The cost for each sheave configuration was obtained from a sheave supplier. The sheave costs were compared to one large sheave of a given diameter in order to determine the cost reduction by using smaller diameter sheaves.

The results of this analysis are presented in tabular form in the Table below. The analysis shows that for each large sheave the number of sheave that would be required is completely impractical. Installing and maintaining 40 to 50 two inch diameter sheaves was not the intent of this pursuit and as such the cost analysis for this configuration was not performed. The Cost savings for the configurations of up to ten - 10 inch diameter sheaves were substantial when compared to the cost of one large sheave.

On closer inspection of the formula in Article 2.5.21 you can see that the actual radius calculated are very small unless the angles are very small or there is little tension in the rope.

The Formula:

$$R = d^2/\ (4.25\sin\theta/2)\sqrt{E/w}$$

For a given rope diameter of 1 inch the wire diameter and 5000 pounds of tension the equation becomes:

$$R = .295/\ (4.25\sin\theta/2)$$

Plugging in different angles and solving for the radius we get:

R	θ
1.75	50
3.42	25
6.81	12.5
13.59	6.25
27.17	3.125
54.34	1.562
108.67	0.781
217.34	0.390

While Article 2.5.21 defines a short arc as an arc less than 50°, no real benefit is gained by this analysis unless the angle is about 3°. A 3° angle of departure is less than the angle of departure of most intermediate deflector sheaves. So AASHTO provides a formula to calculate the effective radius of curvature when turning small angles less than 50° and the effective radius is typically significantly less than the radius of the sheave.

As previously mentioned the AASHTO LRFD Movable Highway Bridge Design Specifications adopted in 2000 has eliminated the effective radius formula. In Article 6.6.5 of the LRFD Specifications the wire rope allowable stresses remains the same as in the Standard Specifications. Article 6.8.3.3.6 of the LRFD Specifications states that: When operating ropes have less than 45 degree arc of contact with a deflector sheave, the minimum sheave diameter shall be 26 times the wire rope diameter. Article 6.8.3.3.4 provides the methodology for determining Wire Rope Stresses.

Formula 6.8.3.3.4-1 calculates the bending stress as:

$\sigma_b = E_W d_W/D$

where:

d_W = diameter of the outer wires in the wire rope
D = tread diameter of the sheave rope grooves

E_W = tensile modulus of elasticity of the steel wire

The maximum total stress in the rope, σ_t may be determined as:

$\sigma_t = P/A + \sigma_b + P_0/A$

where:

P = direct load on ropes
A = effective cross-sectional area of ropes
P_0 = operating loads e.g. the larger of starting or inertial loads

Now if we use these formulas to calculate the rope stresses in a wire rope bent over a deflector sheave through a small angle of 45° or less over a sheave with a diameter of 26 times the rope diameter as allowed by article 6.8.3.3.6 we find that the bending stress in the wire rope exceeds the allowable total tensile stress for direct tension and bending. This is true regardless of the rope diameter. This can be seen by substituting by substituting D = 26 d and dw = .0625 d and solving the equation:

$\sigma_b = E_W d_W/D = E_W .0625d/26d = .0024 E_W$

Where: E_W is 28,500,000 psi

Therefore the bending stress σ_b is equal to 68,510 psi. According to Article 6.6.5 the total tension from direct tension and bending shall not exceed 30 percent of ultimate or 60,900.00 psi. Therefore, if you use a sheave size equal to 26 times the rope diameter, as per Article 6. 6.8.3.3.6, the bending stress calculated in accordance with Article 6.8.3.3.4 will be 12 above the allowable unit tension due to direct tension and bending.

Conclusions

Small diameter sheaves are common place in many industries including cranes, ski lifts, trams, and other haulage applications. The reasons are clear very large diameter sheaves are specially made items that are very expensive. The use of numerous smaller sheaves in some instances can provide significant cost savings. AASHTO provisions have long recognized the use of small diameter sheaves to turns rope through small angle, however the formulas and methods of application are far from clear and in most instances just aren't practical. The AASHTO LRFD Movable Highway Bridge Specification has simplified the approach to small diameter sheaves, however the bending stress calculations provided in the specification indicate that the rope will be always be overstressed if the recommended minimum sheave sizes are used.

More work is required to develop the formulas and methods needed to design the small diameter sheaves. Bending stress and rope fatigue are important concerns that most be given close considerations. However, the potential cost savings to the client cannot be overlooked. New equations should be developed to calculate bending stresses. As we presented, there are many different equations to calculate bending stresses and the range of stresses obtained from these equations vary significantly. Wire rope fatigue calculations consistent with other traditional fatigue analysis methods should be developed that take into account the range of stresses and the number of cycles. For instance operating ropes on a span drive vertical lift bridge experiences more bending cycles than counterweight ropes because of the number of sheaves and drums that the rope passes over for each opening and closing cycle. Also operating ropes experience a larger stress range because the change in direct tension.

We hope that this paper has opened the minds of designers and owners to think outside the box when selecting sheaves for movable bridges.

APPENDIX