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Computer Aided Design/Engineering for Trunnion Girder Design on Water Street Bridge Yahia M. Al-Smadi Parsons

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Introduction

The shrink fit analysis of trunnion girder design can be performed with conventional design methods using conventional shrink fit equations. The assumptions and applications of these equations are limited to simple axisymetrical problems (shaft and desk) regardless of the complexity of the inserted parts. The solution for complex shrink fit assembly can be either performed by boundary element analysis using boundary integral equations for axisymetrical problems or by using cutting edge methods that utilize principles of computer aided design/engineering (CAD/CAE) to analyze the performance of components and assemblies. This paper discus the design procedure and results using conventional shrink fit equations versus CAD/CAE methods (using Solidwork and Cosmos) for the trunnion girder on Eel Pond Bridge.

Project Background

The bridge is a single leaf bascule bridge over Eel Pond on Water Street in Falmouth located in Barnstable County, Massachusetts. The bridge was opened to traffic in 1940 with a total length of 55.1 feet. As of 2003, the average daily traffic was 5,280. It was decided to replace the superstructure and substructure except the footing with a new single leaf bascule span. The new bridge includes two bascule girders mounted to a trunnion girder which rests on two spherical bearings. The lifting machinery is connected to one end of the trunnion girder. The deck and the counterweight are attached to the bascule girders.



Photo 1: Water Street Bridge

Definitions

Trunnion Girder: Trunnion girders can be found in bascule bridges which are supported by bearings outboard of the main bascule girders. A moment connection is required for this type of trunnion. This is usually accomplished by placing a trunnion girder inboard of each main girder and attaching the inner end of the trunnion to it. A simple column is all that is necessary to support the trunnion bearing. The inboard end of the trunnion shaft is usually longer and tapered to a smaller diameter at its end, where it fits tightly into a diaphragm in a cross a girder or beam extending between the main girders (as shown in Figure 1). Trunnion bearings are actually spherical bearings which allow some misalignment of the bearing seat supporting the



trunnion, allowing for some trunnion misalignment, and release of the moment in the trunnion girder [2].

Shrink fitting: Shrink fitting is a procedure in which heat or force fit is used to produce a very strong joint between two parts of metal, one of which is inserted into the other, producing interference and pressure which holds the two parts together mechanically. There are several forms of this type of joint, characterized by the amount of tolerance (space) between the two parts. The shrink fitting process is used for the interference fit type joint [6].

CAD/CAE: CAD is the use of computer technology to aid in the design and especially the drafting (technical drawing and engineering drawing) of a part or product. It is both a visual (or drawing) and symbol-based method of communication whose conventions are particular to a specific technical field. CAE is the use of information technology for supporting engineers in tasks such as analysis, simulation, design, manufacture, planning, diagnosis and repair. Software tools that have been developed for providing support to these activities are considered CAE tools. CAE tools are being used, for example, to analyze the performance of components and assemblies. It encompasses simulation, validation and optimization of products and manufacturing tools [7-8].

Design Models

Three methods can be used for shrink fit analysis depending on the complexity of the assembled parts.

Conventional Shrink Fit Method

Shrink fitting is encountered in many engineering designs. It refers to fitting an object into a slightly smaller cavity. Due to normal forces to the surface that develop at the interface, the inner object shrinks while the outer object expands. The amount of shrinkage or expansion is determined by the material properties as well as the geometry of the components. This fit generates a pressure determined by equations 1 [10] and 2 [5]. The radial and tangential stresses are determined by equations 3[5] and 4[5]. The radial deflection in any part after the assembly shown in equation 5[5], Von Misses Stress and shear stress are shown in equations 6[5] and 7[5]. The equations are shown in Section "Conventional shrink Fit Model".



Boundary Element Method

Boundary element analysis using boundary integral equations (Equation 6) for axisymetrical problems

$$\begin{bmatrix} u_r(p) \\ u_z(p) \end{bmatrix} + 2\pi \int_{\Gamma} \begin{bmatrix} T_{rr}(p,Q) & T_{rz}(p,Q) \\ T_{zr}(p,Q) & T_{zz}(p,Q) \end{bmatrix} \begin{bmatrix} u_r(Q) \\ u_z(Q) \end{bmatrix} r_Q d\Gamma(Q)$$

$$= 2\pi \int_{\Gamma} \begin{bmatrix} U_{rr}(p,Q) & U_{rz}(p,Q) \\ U_{zr}(p,Q) & U_{zz}(p,Q) \end{bmatrix} \begin{bmatrix} t_r(Q) \\ t_z(Q) \end{bmatrix} r_Q d\Gamma(Q)$$
Equation 6

This model takes care of the irregularities and nonlinearities in shrink fit problems better than just the conventional shaft and cylinder. However, it is less efficient than volume-discretisation methods like finite element method (FEM) or finite volume method (FVM). This model requires mathematical and programming expertise that is beyond the scope of this paper. Further discussion can be seen in [3].

CAD/CAE Method

Figure 3 illustrates the CAD model of the bridge where the bascule girders, trunnion girder and machinery are shown. This model is built using Solidworks 3D solid modeling. The Trunnion Girder with the machinery is shown in Figure 4, with only the secondary reducer. A detailed view with all parts and required fits in 2D is shown in Figure 5.



AASHTO Requirements

AASHTO criteria for trunnion girders can be seen by the following subsections of the 1998 and 2000 AASHTO Standard Specifications for Movable Highway Bridges:

3.3.4 Shafts

All shafts shall be straight, true to gage, and turned or otherwise well finished throughout their lengths. All shafts shall be made with fillets where abrupt changes in section occur. All shafts

more than 8 inches in diameter shall have a hole bored lengthwise through the center. The diameter of the hole shall be about 1/5 of the diameter of the shaft.

3.5.2.2 Trunnion Bascule Bridge

Trunnions are supported by trunnion bearings connected either directly or through a trunnion beam to the pier.

6.8.1.3.1 Trunnions

Trunnions shall be designed to transfer span loads to transfer span loads to the trunnion bearings which shall include loads from the span drive machinery during operation. Torsional loading shall be resisted by keys and/or turned bolts. The provisions of Article 6.7.4.1 shall apply to fatigue design. Trunnion design may include distinct hubs that increase the bearing area of the trunnion girders and have hub flanges that bolt to the girder webs to transfer torsional and axial loads. These trunnion hubs shall fit tightly into structural parts with an ANSI/ISO H7/s6 fit and the fit between the hub and trunnion shall be as specified in Table 6.7.8-1. Trunnion designs that do not require hubs shall have an ANSI/ISO H7/s6 fit between the trunnion and mating structural parts. Trunnion collars and/or retaining rings shall be used to transmit torsional and axial loads.

6.8.1.3.2 Trunnion Bearings

Trunnion bearings should be self-aligning spherical plain or rolling element bearings retained in a split steel housing. The bearing assembly shall be designed to support: The dead load and ice load where applicable, live load and impact loads of the bascule span when closed, the dead load and wind loads when open, and a thrust (axial) load equal to approximately 15 percent of the maximum radial load. The bearing housings shall be adjustable to proper elevation, alignment and position on the supporting pedestals in the field by the use of full length shims. The holes through the supporting steel housing for the anchor bolts shall be oversized holes previously drilled in the shop.

Water Street Bridge Trunnion Girder

The Trunnion Girder (Figure 5) on Water Street Bridge is assembled from two trunnion hubs (3) which are interference fit, keyed, and doweled onto two trunnions (1). Two keys^{*} are placed 120 degrees apart on the trunnion (1) and trunnion hub (3) connection. A dowel^{*} helps secure the connection between the trunnion (1) and trunnion hub (3). Two trunnion hubs (3) attach the two bascule girders (4) to the trunnions (1). The trunnions (1) and trunnion bearings (2) are the pivot points about which the bridge rotates. There are three trunnion girders between the two trunnion hubs (3). An outboard trunnion girder (5) is positioned on either side of the center trunnion girder (7). The bascule girder (4) is placed between the outboard trunnion hub (3), bascule girder (4) and the outboard trunnion girder (5). A diaphragm (6) is placed between the outboard trunnion girder (5) and center trunnion girder (7). The smaller end of the trunnion (1) fits into the diaphragm (6). High strength bolts^{*} secure the connection between the diaphragm (6), outboard trunnion girder (5) and center trunnion girder (7).



Trunnion Girder components are symmetrical about the center line

* Parts are not shown in Figure 5 for clarity.

Shrink Fit Analysis

Conventional Shrink Fit Model

Both shrink and clearance fits are used in the trunnion girder components for Water Street Bridge. However, the work in this paper will adjust the fits to be all as shrink fit. Table 1 illustrates the fits between mating parts in the trunnion girder where shrink fit H8/s6 (FN2 fit [4]) is categorized and δ_{Design} is the average of the interference fit limits.

| Shrink | Shaft | Cylinder | Diameter | Fit | Interference Fit | δ _{Design} | AASHTO |
|--------|-------|----------|----------|--------|------------------|---------------------|---------------|
| Fit # | | | | Туре | Limits | _ | Reference |
| 1 | 1 | 2 | 18 | H8/ s6 | 0.0075-0.00116 | 0.00955 | Table 6.7.8-1 |
| 2 | 1 | 3 | 18 | H7/s6 | 0.0075-0.00116 | 0.00955 | Table 6.7.8-1 |
| 3 | 3 | 4 | 28 | H7/s6 | 0.0110-0.0160 | 0.0135 | 6.8.1.3.1 |
| 4 | 3 | 5 | 28 | H7/s6 | 0.0110-0.0160 | 0.0135 | 6.8.1.3.1 |
| 5 | 1 | 6 | 8.875 | H7/s6 | 0.0042-0.0072 | 0.0057 | Table 6.7.8-1 |
| 6 | 6 | 7 | 22 | H7/ s6 | 0.0090-0.0140 | 0.0115 | Table 6.7.8-1 |

Table 1: Illustration of Shrink fit in the Trunnion Girder in Water Street Bridge

All dimensions in inches.

Conventional shrink fit analysis is a well-established field where the objective function is to calculate the shrink fit pressure, deformation and other parameters shown in the subsequent equations. This analysis is based primarily on axisymetrical condition of the engaged parts. In other words, mating parts shall have constant cross section and material during the shrink fit area as shown in Figure 2a. The required equations for the analysis, in their general form, are shown below.

Equation 6

Shrink fit pressure when mating parts have different materials [10]

$${}^{p}\text{Shrink}_{Fit} := \frac{\delta}{2 \cdot b \cdot \left[\frac{1}{E_{0}} \cdot \left(\frac{c^{2} + b^{2}}{c^{2} - b^{2}} + v_{0}\right) + \frac{1}{E_{i}} \cdot \left(\frac{b^{2} + a^{2}}{b^{2} - a^{2}} + v_{i}\right)\right]} \right].$$
 Equation 1

Shrink fit pressure when mating parts have same material [5]

$$p_{Shrink_Fit} := \frac{E \cdot \delta}{b} \cdot \frac{(b^2 - a^2)(c^2 - b^2)}{2 \cdot b^2 (c^2 - a^2)}$$
 Equation 2

$$\sigma_r(r) := \frac{a^2 \cdot p_i - b^2 \cdot p_o}{b^2 - a^2} - \frac{(p_i - p_o) \cdot a^2 \cdot b^2}{b^2 - a^2} \cdot \frac{1}{r^2}$$
Equation 3

$$\sigma_t(r) := \frac{a^2 \cdot p_i - b^2 \cdot p_o}{b^2 - a^2} + \frac{(p_i - p_o) \cdot a^2 \cdot b^2}{(b^2 - a^2) \cdot r^2}$$
Equation 4

$$u(r) := \frac{1 - v}{E} \cdot \frac{\left(a^2 \cdot p_i - b^2 \cdot p_o\right) \cdot r}{b^2 - a^2} + \frac{1 + v}{E} \cdot \frac{\left(p_i - p_o\right) \cdot a^2 \cdot b^2}{\left(b^2 - a^2\right) \cdot r}$$
Equation 5

$$\sigma_{\text{Von}_{Mises}}(\mathbf{r}) \coloneqq \sqrt{\sigma_t(\mathbf{r})^2 + \sigma_r(\mathbf{r})^2 - \sigma_t(\mathbf{r}) \cdot \sigma_r(\mathbf{r})}$$

$$\tau_{\max} := \frac{1}{2} \cdot (\sigma_t - \sigma_r)$$
 . Equation 7

Where

p_{Shrink_Fit}: Pressure due to shrink fit at radius b

- E: Modulus of Elasticity of the steel $29x10^6$ psi [1], assume constant E for all parts except bronze spherical bearing (part (2) in Figure 5) which has $E = 17x10^6$ psi
- δ : δ_{Design} shown in Table 1
- a: inner radius of the shaft as shown in Figure 2
- b: outer radius of the shaft and inner radius of the cylinder as shown in Figure 2
- c: outer radius of the cylinder as shown in Figure 2
- r: arbitrary distance between a and b or between b and c
- $\sigma_r(r)$: radial stress in the shaft as function of r. For the cylinder replace a and b with b and c
- $\sigma_t(r)$: tangential stress in the shaft as function of r. For the cylinder replace a and b with b and c
- u(r) : radial displacement in the shaft at any arbitrary distance. For the cylinder replace a and b with b and c

v: Poisson ratio 0.29 for steel [9] τ_{max} : maximum shear stress

Note that axial loading is absent (ends are open or unconstrained) and therefore $\sigma_z = 0$ which means that the stresses are clearly symmetrical about the z axis and deformations likewise which leads to cancel tangential displacement in the symmetrical field. δ_{Design} shown in Table 1 is assumed in the middle of the interference fit limits. FEA takes different approach to determine δ_{Design} which depends on volume integral of the material around shrink fit area. Shrink fit pressure will act equally on both parts causing the outer radius of the shaft to decrease and inner radius of the cylinder to increase to

$$\delta := \frac{b \cdot p \text{ Shrink_fit}}{E_0} \cdot \left(\frac{b^2 + c^2}{c^2 - b^2} + v_0 \right) + \frac{b \cdot p \text{ Shrink_fit}}{E_i} \cdot \left(\frac{a^2 + b^2}{b^2 - a^2} - v_i \right)$$

This shrink allowance is a function of modulus of elasticity, poisson ratio and geometry of the mating parts. Table 2 shows the result of applying equations 1 through 6 after specifying the appropriate boundary conditions for each shrink fit case.

| 1 4010 21 | | Tor the fram | | i ater su | eer Briage | | | | | |
|--|---------------------|----------------|-------------------------|---------------|---------------|--------|-----------------------------|--------------|--|--|
| Shrink | | Inch | p _{Shrink_Fit} | $\sigma_r(r)$ | $\sigma_t(r)$ | u(r) | $\sigma_{\text{Von Mises}}$ | τ_{max} | | |
| Fit # | | men | (psi) | (psi) | (psi) | (inch) | (psi) | (psi) | | |
| | | | | | | | | | | |
| | Trunnian (1) | a = 0.875 | $p_i = 0$ | 0 | -5254 | 0 | 5254 | -2627 | | |
| 1 | | b = 9 | $p_{o} = 2602$ | -2602 | -2652 | -0.001 | 2627 | -25 | | |
| 1 | Dearing (2) | b = 9 | $p_i = 2602$ | -2602 | 6268 | 0.004 | 7897 | 4435 | | |
| | Bearing (2) | $c = 14^{(1)}$ | $p_{\rm o}=0$ | 0 | 3666 | 0.003 | 3666 | 1833 | | |
| ⁽¹⁾ See trunnion bearing (Figure 6) explanation | | | | | | | | | | |
| | Trunnion (1) | a = 0 | $p_i = 0$ | 0 | -9072 | 0 | 9072 | -4536 | | |
| 2 | | b = 9 | $p_{\rm o} = 4493$ | -4493 | -4579 | -0.001 | 4537 | -43 | | |
| Δ | | b = 9 | $p_i = 4493$ | -4493 | -13215 | -0.004 | 11638 | -4361 | | |
| | Hub (3) | $c = 14^{(2)}$ | $p_{\rm o}=7052$ | -7052 | -10656 | -0.004 | 9388 | -1802 | | |
| ⁽²⁾ See tr | unnion hub (Figure | e 7) explanati | on | | | | | | | |
| | Hub (3) | a = 9 | $p_i = 4493$ | -4493 | -13215 | -0.004 | 11638 | -4361 | | |
| 2 | | b = 14 | $p_{\rm o}=7052$ | -7052 | -10656 | -0.004 | 9388 | -1802 | | |
| 5 | | b = 14 | $p_i = 7052$ | -7052 | 10978 | 0.011 | 15738 | 9015 | | |
| | B. Girder (4) | $c = 30^{(3)}$ | $p_{\rm o}=0$ | 0 | 3927 | 0.007 | 3927 | 1963 | | |
| ⁽³⁾ See bascule girder explanation | | | | | | | | | | |
| | Hub (3) | a = 9 | $p_i = 4493$ | -4493 | -8208 | -0.002 | 7119 | -1857 | | |
| 4 | | b = 14 | $p_{\rm o} = 5583$ | -5583 | -7118 | -0.003 | 6488 | -768 | | |
| 4 | | b = 14 | $p_i = 5583$ | -5583 | 14516 | 0.013 | 17970 | 10050 | | |
| | O. T. Girder (5) | $c = 21^{(4)}$ | $p_{\rm o}=0$ | 0 | 8933 | 0.011 | 8933 | 4467 | | |
| ⁽⁴⁾ See or | utboard trunnion gi | irder (Figure | 8) explanation | | | | | | | |
| | Trunnion (1) | a = 0 | $p_i = 0$ | 0 | -30781 | -0.001 | 30781 | -15390 | | |
| 5 | | b = 4.5 | $p_{\rm o} = 14808$ | -14808 | -15972 | -0.002 | 15423 | -582 | | |
| 5 | | b = 4.5 | $p_i = 14808$ | -14808 | -2297 | 0 | 13804 | 6256 | | |
| | Diaphragm (2) | $c = 11^{(5)}$ | $p_{\rm o} = 9600$ | -9600 | -7506 | -0.002 | 8743 | 1047 | | |
| ⁽⁵⁾ See di | aphragm (Figure 9 |) explanation | <u>1</u> | | | | | | | |
| | Diaphragm (6) | a = 4.5 | $p_i = 14808$ | -14808 | -2297 | 0 | 13804 | 6256 | | |
| 6 | | b = 11 | $p_{o} = 9600$ | -9600 | -7506 | -0.002 | 8743 | 1047 | | |
| 0 | | b = 11 | $p_i = 9600$ | -9600 | 16860 | 0.013 | 23200 | 13230 | | |
| | C.T. Girder (7) | $c = 21^{(5)}$ | $p_{0} = 0$ | 0 | 7260 | 0.009 | 7260 | 3630 | | |

Table 2: Shrink Fit Results for the Trunnion Girder on Water Street Bridge

⁽¹⁾ Trunnion bearing (2) is a spherical bushing inside spherical housing with a cross section as shown in Figure 6 Where $r_1 < r_2 < r_3$. Due to limitations of the conventional model, the bearing is considered cylindrical with inner radius r_1 equal to 9 inches and outer radius r_3 equal to 14 inches.



⁽²⁾ Trunnion hub (3) as shown in Figure 7 is also considered cylindrical due to the limitation of the conventional model. Note that the geometry of the hub includes many features (stress concentration areas) such as fillets, keyways, dowel pin hole, mounting bolt holes and stiffeners which make it impossible for conventional shrink fit model to solve such complex



the trunnion at r_1 , and outside pressure due to shrink fit with bascule girder and outboard trunnion girder at r_4 . It is expected that the trunnion hub outside diameter r_4 will grow due to trunnion shrink fit, so adjustment to the bore of bascule girder and outboard trunnion girder shall be made prior to their fabrication.

⁽³⁾ Bascule girder (Figures 3 and 5) is the biggest part the dimension of the outside diameter was assumed. Unfortunately, that is how the conventional shrink fit model works. It assumes uniform simple cylinder geometry.

⁽⁴⁾ Outboard and center trunnion girders are made of two disk shaped flanges welded to the ends of a tube. This structure helps prevent or reduce radial displacement when the shrink fit occurs at the bore of the flange. Radial and tangential stresses also exist in the tube and will be ignored due to the limitations of the conventional shrink fit model.





The tangential and radial stresses distribution for each part varies according to the side in which the pressure is applied (i.e. internal or external pressure). The trunnion, for example, is subject to external pressure only. The trunnion hub and diaphragm are subject to internal and external pressures. Finally, the bascule girder and center and outboard trunnion girders are subject to internal pressure only. Tangential and radial stresses are normalized with respect to shrink fit pressure as shown in Figures 10 through 12. Figure 10 illustrates the distribution of tangential stress (dotted line) and radial stress (solid line) in the mating parts that are subject to external pressure only such as the trunnion. The radial stress is minimum at the inner radius while the maximum magnitude of the circumferential stress (tangential stress) occurs at the inside surface of the trunnion.



Figure 11 illustrates the distribution of tangential stress (dotted line) and radial stress (solid line) in the mating parts that are subject to internal pressure only such as the bascule girder, center and outboard trunnion girders. Both stresses decrease as the radius increase. Radial stress becomes negligible at the outer surface.



Figure 12 illustrates the distribution of tangential stress (dotted line) and radial stress (solid line) in the mating parts that are subject to internal and external pressure such as trunnion hub and diaphragm.

CAD/CAE Shrink Fit Model

The analysis for any model follows three brief stages. The first stage is preprocessing where the material for all components is defined, loads and constraints are applied, contacts and meshing properties are specified. Then the processing stage starts which is actually running the analysis. Once the analysis is completed the third stage starts which is extracting all the results from the analysis. SolidWorks is used as CAD to create the solid model in 3D (Figures 3, 4 and 13), Cosmos Designer which is integrated with SolidWorks and used as CAE platform to perform the shrink fit analysis using FEA techniques.

Creating the model.

The model is drawn in 1D, 2D or 3D space in the appropriate units (m, mm, in, etc.). The model may be created in the preprocessor, or it can be imported from another CAD software package via direct translation or through a neutral file format (IGES, STEP, ACIS, Parasolid, DXF, etc.). SolidWorks is a 3D solid modeling environment where parts can be drawn in 2D and extruded to the 3D final shape. The Trunnion Girder model ia shown in Figure 13 as isometric view and isometric cross section view. The interferences of the assembled parts are described in Figure 5.



Defining the Material.

Figure 14 demonstrates the material selection for all parts except the bearing which is shown in Figure 15. These material properties are defined by default in CosmosWorks library. However, the designer has an ability to change or customize existing material as well as create new material properties table as shown in the below table (Figure 14 and 15).

| Property | Description | Value | Units | Property | Description | Value | Units |
|----------|----------------------|------------------|--------------|-----------------------|----------------------|----------------|--------------|
| EX | Elastic modulus | 29007547.55 | psi | B Berullium Co | Elastic modulus | 14503773.77 | psi |
| NUXY | Poisson's ratio | 0.29 | NA | B Berullium Co NUXY | Poisson's ratio | 0.33 | NA |
| GXY | Shear modulus | 11167905.81 | psi | B Berullium Co | Shear modulus | 5366396.296 | psi |
| DENS | Mass density | 0.2854056554 | lb/in^3 | Berullium S.: DENS | Mass density | 0.3070820343 | lb/in^3 |
| SIGXT | Tensile strength | 60989.38398 | psi | B Berullium S-f SIGXT | Tensile strength | 69387.93922 | psi |
| SIGXC | Compressive strength | | psi | B Brass SIGXC | Compressive strength | | psi |
| SIGYLD | Yield strength | 50991.06249 | psi | B Chromium Cu SIGYLD | Yield strength | 34763.95032 | psi |
| ALPX | Thermal expansion co | 8.333333333e-006 | /Fahrenheit | Commercial ALPX | Thermal expansion co | 1e-005 | /Fahrenheit |
| KX | Thermal conductivity | 0.0006286132556 | BTU/(in.s.F) | B Copper KX | Thermal conductivity | 0.001471222513 | BTU/(in.s.F) |
| C | Specific heat | 0.1003344482 | Btu/(lb.F) | Copper-Cob | Specific heat | 0.09316770186 | Btu/(Ib.F) |

Shrink fit analysis in Cosmos Designer requires the designer to define the contact surface. The designer shall have the adequate knowledge in nonlinear analysis, because the presence of contact surfaces leads the FEA analysis to be nonlinear. Nonlinear runs take longer to setup and solve than similar linear analysis, so the decision to use nonlinear analysis should derived from a need that can not be satisfied by a linear approximation. Other than two surfaces in contact considered as nonlinear, a problem becomes nonlinear if the loading causes significant changes in the structure stiffness or stress levels approach the Yield Point – Strains beyond the elastic limit (plasticity) or large displacements occur or are expected.

Figure 16 describes the nonlinear sources in the analysis. The Trunnion Girder has least two sources of nonlinearities, these are the geometric and change of status nonlinearities. In general, analysis of metal forming, crash and impact and elastomers and polymers can be considered as good examples for non linear analysis.





• Defining the element type.

Cosmos Designer performs solid analysis based on elliptical tetrahedral element. This solid element will have 10 nodes: 4 corner nodes and one node at the middle of each edge (a total of six mid-side nodes). Meshing in Cosmos Designer can generate 3D tetrahedral solid elements, and 2D triangular shell elements as well as mixing the two types of elements in studies created with the mixed mesh type. Shell elements are naturally suitable for modeling thin parts (sheet metals).



• Defining the model mesh properties.

Meshing is a crucial step in design analysis. Global and local meshes are available in Cosmos Designer for components, faces, edges, and vertices. Global mesh is based on a global element size, tolerance, while the local mesh controls element size for the specified entities, element growth ratio and number of layers of elements. The size of the generated mesh (number of nodes and elements) depends on the geometry and dimensions of the model, element size, mesh tolerance, mesh control, and contact specifications. In the early stages of design analysis where approximate results may suffice, designer can specify a larger element size for a faster solution.



For a more accurate solution, a smaller element size may be required. Figure 17 shows the model with coarse mesh and finer mesh.

Results

After completing of the model and setup of the shrink fit contacts Direct Sparse was chosen to be the solver in Cosmos Designer. Inertial relief, soft spring and other shrink fit options were also activated prior to the analysis. Stresses such as $\sigma_{Von Mises}$, σ_r , σ_t , u(r), $\tau_{xy} \tau_{xz} \tau_{yz}$, and p_{Shrink_Fit} , deflections such as u_{Total} , u_r , u_t , u(r), u_z and other deflection properties can be extracted from the result folder in Cosmos Designer. Some of the results are shown in the figures below.









Conclusion

A study of shrink fit for the trunnion girder on Water Street Bridge was investigated using two models. Conventional model and CAD/CAE model. The CAE model was performed using Cosmos Designer with a mesh element size of 3.5 inch. A finer mesh can also be used and mesh control can be applied to attain more accurate results. Rigid body load and external loads can be added to CAE model for better understanding of the stress distribution. deformation distribution, fatigue and vibration modes in the trunnion girder. The designer must not over simplify the model during the CAD stage and should also be aware of the boundary and loading conditions during the preprocessing



and loading conditions during the preprocessing Photo 2: Trunnion Girder on Water Street Bridge stage. A complete fabricated trunnion girder is shown in photo 2.

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APPENDIX







Shrink Fit Anaylsis of Trunnion and Trunnion Bearing



Shrink Fit Pressure

a := 0.875 in b := 9in c := 14 in 5 := 0.00955 in

v_i := 0.29 v_o :=

Figure 1: Press fit assembly a) Before Press Fit b) After Press Fit

$$p_{\text{Shrink}}_{\text{Fit}} := \frac{5}{2 \cdot b \left[\frac{1}{E_0} \cdot \left(\frac{c^2 + b^2}{c^2 - b^2} + v_0 \right) + \frac{1}{E_0} \cdot \left(\frac{b^2 + a^2}{b^2 - a^2} + v_1 \right) \right]} \right]}$$

$$p_{\text{Shrink}}_{\text{Fit}} := 2602 \text{ psi}$$

Stresses in the Trunnion

| r := a | $p_i := 0p_{si}$ $p_o := p_{Shrink_Fit}$ | | | |
|---|--|---|--|---|
| $\sigma_{\underline{r}}(\underline{r}) := \frac{a^2 \cdot p_{\underline{i}} - b^2 \cdot p_{\underline{o}}}{b^2 - a^2} - \frac{(p_{\underline{i}} - p_{\underline{o}}) \cdot a^2}{b^2 - a^2}$ | $\frac{b^2}{r^2} \cdot \frac{1}{r^2} \qquad \qquad \sigma_t(r) := \frac{a^2 \cdot p_i - b^2 \cdot p_o}{b^2 - a^2} + \frac{(p_i - p_o)}{(b^2 - a)^2}$ | $\frac{a^2 \cdot b^2}{E_i} \qquad u(x) := \frac{1 - v_i}{E_i} \frac{\left(a^2 \cdot p_i - b^2 \cdot p_o\right) \cdot r}{b^2 - a^2} + \frac{1 + v_i}{E_i}$ | $\frac{\left(\mathbf{p}_{1}-\mathbf{p}_{0}\right)\cdot\mathbf{a}^{2}\cdot\mathbf{b}^{2}}{\left(\mathbf{b}^{2}-\mathbf{a}^{2}\right)\cdot\mathbf{r}} \qquad \sigma_{\mathrm{Von}_\mathrm{Mises}}(\mathbf{r}):=\sqrt{\sigma_{t}(\mathbf{r})^{2}+\sigma_{t}(\mathbf{r})^{2}-\sigma_{t}(\mathbf{r})\cdot\sigma_{t}(\mathbf{r})}$ | $\tau_{max}(\mathbf{r}) \coloneqq 0.5 \big(\sigma_t(\mathbf{r}) - \sigma_r(\mathbf{r})\big)$ |
| $\sigma_{\rm r}({\bf r})=0{\rm psi}$ | $\sigma_t(\mathbf{r}) = -5254 \mathrm{psi}$ | u(r) = -0.0002 in | $\sigma_{\text{Von}_{\text{Mises}}}(\mathbf{r}) = 5254 \text{psi}$ | $\tau_{\max}(\mathbf{r}) = -2627 \mathrm{psi}$ |
| $\begin{aligned} \mathbf{r} &:= \mathbf{b} \\ \sigma_{\mathbf{r}}(\mathbf{r}) &:= \frac{a^2 \cdot \mathbf{p}_1 - b^2 \cdot \mathbf{p}_0}{b^2 - a^2} - \frac{(\mathbf{p}_1 - \mathbf{p}_0) \cdot a^2}{b^2 - a^2} \\ \sigma_{\mathbf{r}}(\mathbf{r}) &= -2602 \text{psi} \end{aligned}$ | $\frac{b^2}{r^2} \cdot \frac{1}{r^2} \qquad \qquad \sigma_t(r) := \frac{a^2 \cdot p_i - b^2 \cdot p_o}{b^2 - a^2} + \frac{(p_i - p_o)}{(b^2 - a)^2}$ $\sigma_t(r) = -2652 \text{ psi}$ | $\frac{(a^2 \cdot b^2)}{2} \qquad u(r) := \frac{1 - v_i}{E_i} \cdot \frac{(a^2 \cdot p_i - b^2 \cdot p_o) \cdot r}{b^2 - a^2} + \frac{1 + v_i}{E_i}$ $u(r) = -0.0006 \text{ in}$ | $ \begin{array}{l} \displaystyle \frac{(\mathbf{p}_{1}-\mathbf{p}_{0})\cdot\mathbf{a}^{2}\cdot\mathbf{b}^{2}}{\left(\mathbf{b}^{2}-\mathbf{a}^{2}\right)\cdot\mathbf{r}} & \sigma_{\mathrm{Von}_\mathrm{Mises}}(\mathbf{r}) \coloneqq \sqrt{\sigma_{t}(\mathbf{r})^{2}+\sigma_{t}(\mathbf{r})^{2}-\sigma_{t}(\mathbf{r})\cdot\sigma_{t}(\mathbf{r})} \\ \sigma_{\mathrm{Von}_\mathrm{Mises}}(\mathbf{r}) \equiv 2627\mathrm{psi} \end{array} $ | $\begin{split} \tau_{\max}(\mathbf{r}) &:= 0.5 \big(\sigma_t(\mathbf{r}) - \sigma_t(\mathbf{r}) \big) \\ \tau_{\max}(\mathbf{r}) &= -25 \text{psi} \end{split}$ |
| Stresses in the Spherical | Trunnion Bearing k Fit po := 0psi | | | |
| $\sigma_{\mathbf{r}}(\mathbf{r}) := \frac{\mathbf{b}^2 \cdot \mathbf{p}_1 - \mathbf{c}^2 \cdot \mathbf{p}_0}{\mathbf{c}^2 - \mathbf{b}^2} - \frac{(\mathbf{p}_1 - \mathbf{p}_0) \cdot \mathbf{b}^2}{\mathbf{c}^2 - \mathbf{b}^2}$ $\sigma_{\mathbf{r}}(\mathbf{r}) = -2602\mathrm{psi}$ | $\sigma_{t}(r) := \frac{b^{2} \cdot p_{i} - c^{2} \cdot p_{o}}{c^{2} - b^{2}} + \frac{(p_{i} - p_{o})}{(c^{2} - b)}$ $\sigma_{t}(r) = 6268 \text{ psi}$ | $\frac{(b^2 \cdot c^2)}{(2) \cdot r^2} \qquad u(r) := \frac{1 - v_o}{E_o} \cdot \frac{(b^2 \cdot p_1 - c^2 \cdot p_o) \cdot r}{c^2 - b^2} + \frac{1 + v_o}{E_o}$ $u(r) = 0.0037 \text{ in}$ | $\frac{(\mathbf{p}_{i} - \mathbf{p}_{o}) \cdot \mathbf{b}^{2} \cdot \mathbf{c}^{2}}{(\mathbf{c}^{2} - \mathbf{b}^{2}) \cdot \mathbf{r}} \qquad \sigma_{\text{Von}_{i} \text{Mises}}(\mathbf{r}) \coloneqq \sqrt{\sigma_{t}(\mathbf{r})^{2} + \sigma_{t}(\mathbf{r})^{2} - \sigma_{t}(\mathbf{r}) \cdot \sigma_{r}(\mathbf{r})}$ $\sigma_{\text{Von}_{i} \text{Mises}}(\mathbf{r}) = 7897 \text{ psi}$ | $\begin{split} \tau_{\max}(r) &:= 0.5 \Big(\sigma_t(r) - \sigma_t(r) \Big) \\ \tau_{\max}(r) &= 4435 \text{psi} \end{split}$ |
| r := c | | | | |
| $\sigma_{\underline{r}}(\underline{r}) := \frac{\underline{b}^2 \cdot \underline{p}_{\underline{i}} - \underline{c}^2 \cdot \underline{p}_{\underline{o}}}{\underline{c}^2 - \underline{b}^2} - \frac{\left(\underline{p}_{\underline{i}} - \underline{p}_{\underline{o}}\right) \cdot \underline{b}^2}{\underline{c}^2 - \underline{b}^2}$ | $\frac{\cdot c^2}{\cdot r^2} \cdot \frac{1}{r^2} \qquad \qquad \sigma_t(x) := \frac{b^2 \cdot p_i - c^2 \cdot p_o}{c^2 - b^2} + \frac{(p_i - p_o)}{(c^2 - b)}$ | $\frac{\mathbf{b} \cdot \mathbf{b}^2 \cdot \mathbf{c}^2}{\mathbf{c}^2} \qquad \mathbf{u}(\mathbf{r}) \coloneqq \frac{1 - \mathbf{v}_{\mathbf{o}}}{\mathbf{E}_{\mathbf{o}}} \cdot \frac{\left(\mathbf{b}^2 \cdot \mathbf{p}_{\mathbf{i}} - \mathbf{c}^2 \cdot \mathbf{p}_{\mathbf{o}}\right) \cdot \mathbf{r}}{\mathbf{c}^2 - \mathbf{b}^2} + \frac{1 + \mathbf{v}_{\mathbf{o}}}{\mathbf{E}_{\mathbf{o}}}$ | $\frac{(\mathbf{p_i} - \mathbf{p_o}) \cdot \mathbf{b}^2 \cdot \mathbf{c}^2}{(\mathbf{c}^2 - \mathbf{b}^2) \cdot \mathbf{r}} \qquad \sigma_{\text{Von_Mises}}(\mathbf{r}) \coloneqq \sqrt{\sigma_t(\mathbf{r})^2 + \sigma_t(\mathbf{r})^2 - \sigma_t(\mathbf{r}) \cdot \sigma_t(\mathbf{r})}$ | $\tau_{max}(\mathbf{r}) \coloneqq 0.5 \big(\sigma_t(\mathbf{r}) - \sigma_r(\mathbf{r}) \big)$ |
| $\sigma_{v}(\mathbf{r}) = 0 \text{ psi}$ | $\sigma_t(\mathbf{r}) = 3666 \mathrm{psi}$ | u(r) = 0.003 in | σ _{Von Mises} (r) = 3666 psi | $\tau_{max}(r) = 1833 psi$ |

Shrink Fit Anaylsis of Trunnion and Trunnion Bearing



Shrink Fit Pressure

a := 0.875 · in b := 9 in c := 14 · in δ := 0.00955 in

 ${}^{p} \mathrm{Shrink_Fit} \stackrel{:=}{ } \frac{ \delta }{ 2 \cdot b \Biggl[\frac{1}{E_o} \cdot \Biggl(\frac{c^2 + b^2}{c^2 - b^2} + v_o \Biggr) + \frac{1}{E_i} \cdot \Biggl(\frac{b^2 + a^2}{b^2 - a^2} + v_i \Biggr) \Biggr] } .$

p_{Shrink_Fit} = 2602 psi

Figure 1: Press fit assembly a) Before Press Fit b) After Press Fit

Stresses in the Trunnion

| $\mathbf{r} := \mathbf{a}$ | $\mathbf{p}_i \coloneqq 0\mathbf{p}_i = \mathbf{p}_{shrink}_{rit}$ | | | |
|---|---|---|--|---|
| $\sigma_{\mathbf{r}}(\mathbf{r}) := \frac{\mathbf{a}^2 \cdot \mathbf{p}_{\mathbf{i}} - \mathbf{b}^2 \cdot \mathbf{p}_{\mathbf{o}}}{\mathbf{b}^2 - \mathbf{a}^2} - \frac{(\mathbf{p}_{\mathbf{i}} - \mathbf{p}_{\mathbf{o}}) \cdot \mathbf{a}^2}{\mathbf{b}^2 - \mathbf{a}^2}$ | $\frac{b^2}{r^2} \cdot \frac{1}{r^2} \qquad \qquad \sigma_t(r) := \frac{a^2 \cdot \mathbf{p}_1 - b^2 \cdot \mathbf{p}_0}{b^2 - a^2} + \frac{(\mathbf{p}_1 - \mathbf{p}_0) \cdot a^2 \cdot b^2}{(b^2 - a^2) \cdot r^2}$ | $\mathbf{u}(\mathbf{r}) := \frac{1 - v_{1}}{E_{1}} \cdot \frac{\left(\mathbf{a}^{2} \cdot \mathbf{p}_{1} - \mathbf{b}^{2} \cdot \mathbf{p}_{0}\right) \cdot \mathbf{r}}{\mathbf{b}^{2} - \mathbf{a}^{2}} + \frac{1 + v_{1}}{E_{1}} \cdot \frac{\left(\mathbf{p}_{1} - \mathbf{p}_{0}\right) \cdot \mathbf{a}^{2} \cdot \mathbf{b}^{2}}{\left(\mathbf{b}^{2} - \mathbf{a}^{2}\right) \cdot \mathbf{r}}$ | $\sigma_{Von_Mites}(\mathbf{r}) \coloneqq \sqrt{\sigma_t(\mathbf{r})^2 + \sigma_t(\mathbf{r})^2 - \sigma_t(\mathbf{r}) \cdot \sigma_t(\mathbf{r})}$ | $\tau_{max}(\mathbf{r}) \coloneqq 0.5 \big(\sigma_t(\mathbf{r}) - \sigma_t(\mathbf{r})\big)$ |
| $\sigma_{\rm r}(r)=0{\rm psi}$ | $\sigma_t(\mathbf{r}) = -5254 \mathrm{psi}$ | u(z) = -0.0002 in | $\sigma_{\text{Von}_{\text{Mises}}(r)} = 5254 \text{psi}$ | $\tau_{\max}(\mathbf{r}) = -2627\mathrm{psi}$ |
| $\begin{aligned} \mathbf{r} &:= \mathbf{b} \\ \sigma_{\mathbf{z}}(\mathbf{r}) &:= \frac{\mathbf{a}^2 \cdot \mathbf{p}_1 - \mathbf{b}^2 \cdot \mathbf{p}_0}{\mathbf{b}^2 - \mathbf{a}^2} - \frac{(\mathbf{p}_1 - \mathbf{p}_0) \cdot \mathbf{a}^2}{\mathbf{b}^2 - \mathbf{a}^2} \\ \sigma_{\mathbf{z}}(\mathbf{r}) &= -2602 \text{psi} \end{aligned}$ | $\frac{b^2}{r^2} \cdot \frac{1}{r^2} \qquad \qquad \sigma_t(r) := \frac{a^2 \cdot p_i - b^2 \cdot p_o}{b^2 - a^2} + \frac{(p_i - p_o) \cdot a^2 \cdot b^2}{(b^2 - a^2) \cdot r^2} \\ \sigma_t(r) = -2652 p_{Si}$ | $\begin{split} u(r) &:= \frac{1 - v_i}{E_i} \cdot \frac{\left(a^2 \cdot p_i - b^2 \cdot p_o\right) \cdot r}{b^2 - a^2} + \frac{1 + v_i}{E_i} \cdot \frac{\left(p_i - p_o\right) \cdot a^2 \cdot b^2}{\left(b^2 - a^2\right) \cdot r} \\ u(r) &= -0.0006 \text{ in} \end{split}$ | $\begin{split} \sigma_{\mathrm{Von}_\mathrm{Mises}}(\mathbf{r}) & \coloneqq \sqrt{\sigma_t(\mathbf{r})^2 + \sigma_t(\mathbf{r})^2 - \sigma_t(\mathbf{r}) \cdot \sigma_t(\mathbf{r})} \\ \sigma_{\mathrm{Von}_\mathrm{Mises}}(\mathbf{r}) & = 2627\mathrm{psi} \end{split}$ | $\begin{split} \tau_{\max}(\mathbf{r}) &:= 0.5 \big(\sigma_t(\mathbf{r}) - \sigma_t(\mathbf{r})\big) \\ \tau_{\max}(\mathbf{r}) &= -25\mathrm{psi} \end{split}$ |
| Stresses in the Spherical | Trunnion Bearing k Fit po := 0psi | | | |
| $\sigma_{r}(r) := \frac{b^{2} \cdot p_{1} - c^{2} \cdot p_{0}}{c^{2} - b^{2}} - \frac{(p_{1} - p_{0}) \cdot b^{2}}{c^{2} - b^{2}}$ $\sigma_{r}(r) = -2602 psi$ | $ \begin{array}{c} -\frac{\mathbf{c}^2}{\mathbf{c}^2} \cdot \frac{1}{\mathbf{r}^2} & \sigma_t(\mathbf{r}) \coloneqq \frac{\mathbf{b}^2 \cdot \mathbf{p}_1 - \mathbf{c}^2 \cdot \mathbf{p}_0}{\mathbf{c}^2 - \mathbf{b}^2} + \frac{(\mathbf{p}_1 - \mathbf{p}_0) \cdot \mathbf{b}^2 \cdot \mathbf{c}^2}{(\mathbf{c}^2 - \mathbf{b}^2) \cdot \mathbf{r}^2} \\ \sigma_t(\mathbf{r}) = 6268 \mathrm{psi} \end{array} $ | $\begin{split} \mathbf{u}(\mathbf{r}) &\coloneqq \frac{1 - v_o}{E_o} \cdot \frac{\left(\mathbf{b}^2 \cdot \mathbf{p}_i - \mathbf{c}^2 \cdot \mathbf{p}_o\right) \cdot \mathbf{r}}{\mathbf{c}^2 - \mathbf{b}^2} + \frac{1 + v_o}{E_o} \cdot \frac{\left(\mathbf{p}_i - \mathbf{p}_o\right) \cdot \mathbf{b}^2 \cdot \mathbf{c}^2}{\left(\mathbf{c}^2 - \mathbf{b}^2\right) \cdot \mathbf{r}} \\ \mathbf{u}(\mathbf{r}) &= 0.0037 \text{ in} \end{split}$ | $\begin{split} \sigma_{\mathrm{Von}_\mathrm{Mises}}(\mathbf{r}) &\coloneqq \sqrt{\sigma_{\mathrm{t}}(\mathbf{r})^{2} + \sigma_{\mathrm{r}}(\mathbf{r})^{2} - \sigma_{\mathrm{t}}(\mathbf{r}) \cdot \sigma_{\mathrm{r}}(\mathbf{r})} \\ \sigma_{\mathrm{Von}_\mathrm{Mises}}(\mathbf{r}) &= 7897 \mathrm{psi} \end{split}$ | $\begin{split} \tau_{\max}(r) &:= 0.5 \big(\sigma_t(r) - \sigma_t(r)\big) \\ \tau_{\max}(r) &= 4435 \text{psi} \end{split}$ |
| r := c | | | | |
| $\sigma_{\underline{r}}(\underline{r}) := \frac{\underline{b}^2 \cdot \underline{p}_{\underline{i}} - \underline{c}^2 \cdot \underline{p}_{\underline{o}}}{\underline{c}^2 - \underline{b}^2} - \frac{(\underline{p}_{\underline{i}} - \underline{p}_{\underline{o}}) \cdot \underline{b}^2}{\underline{c}^2 - \underline{b}^2}$ | $\frac{\mathbf{c}^2}{\mathbf{r}^2} \cdot \frac{1}{\mathbf{r}^2} \qquad \qquad \boldsymbol{\sigma}_t(\mathbf{r}) \coloneqq \frac{\mathbf{b}^2 \cdot \mathbf{p}_i - \mathbf{c}^2 \cdot \mathbf{p}_o}{\mathbf{c}^2 - \mathbf{b}^2} + \frac{(\mathbf{p}_i - \mathbf{p}_o) \cdot \mathbf{b}^2 \cdot \mathbf{c}^2}{(\mathbf{c}^2 - \mathbf{b}^2) \cdot \mathbf{r}^2}$ | $u(\mathbf{r}) \coloneqq \frac{1-v_o}{E_o} \cdot \frac{\left(\mathbf{b}^2 \cdot \mathbf{p}_1 - \mathbf{c}^2 \cdot \mathbf{p}_o\right) \cdot \mathbf{r}}{\mathbf{c}^2 - \mathbf{b}^2} + \frac{1+v_o}{E_o} \cdot \frac{\left(\mathbf{p}_1 - \mathbf{p}_o\right) \cdot \mathbf{b}^2 \cdot \mathbf{c}^2}{\left(\mathbf{c}^2 - \mathbf{b}^2\right) \cdot \mathbf{r}}$ | $\sigma_{Von_Mises}(\mathbf{r}) \coloneqq \sqrt{\sigma_t(\mathbf{r})^2 + \sigma_t(\mathbf{r})^2 - \sigma_t(\mathbf{r}) \cdot \sigma_t(\mathbf{r})}$ | $\tau_{max}(\mathbf{r}) \coloneqq 0.5 \big(\sigma_t(\mathbf{r}) - \sigma_t(\mathbf{r}) \big)$ |
| $\sigma_{\rm r}({\rm r}) = 0{\rm psi}$ | $\sigma_t(\mathbf{r}) = 3666 \mathrm{psi}$ | u(r) = 0.003 in | σ _{Von Mises} (r) = 3666 psi | $\tau_{max}(r) = 1833 psi$ |

Shrink Fit Anaylsis of Trunnion, Trunnion Hub, and Bascule Girder

| Shrink Fit Anaylsis of Trunnion and Trunnion Hub | 1 | (2, 2)(2, 2) | |
|--|--|---|---|
| $E := 29000000 \text{psi} v := 0.29 a := 0.875 \text{in} b := 9 \text{in} \qquad c := 14 \text{in} \qquad \delta := 0.00478 \text{in}$ | Shrink Fit Pressure | $PShrink_Fit_i := \frac{E \cdot \delta}{b} \cdot \frac{(b^2 - a^2)(c^2 - b^2)}{c + 2(c^2 - b^2)}$ $PShrink_Fit_i$ | _i = 4493 psi |
| Shrink Fit Anaylsis of Trunnion Hub and Bascule Gird | er | 2-b (c - a) | |
| $E := 29000000 p_{\text{si}} \ v := 0.29 \ a := 9 \text{in} \ b := 14 \text{in} \ c := 30 \text{in} \ \delta := 0.0135 \text{in}$ | Shrink Fit Pressure | $p_{Shrink}_{it_o} := \frac{E \cdot 5}{b} \cdot \frac{(b^2 - a^2)(c^2 - b^2)}{2 \cdot b^2(c^2 - a^2)} $ $p_{Shrink}_{it_o}$ | o = 7052 psi |
| Stresses in the Trunnion | | | |
| $a:=0.875 \text{in} b:=9 \text{in} c:=14 \text{in} \qquad p_1:=0 \text{psi} \qquad p_o:=p_{Shrink}_{ii_1} \text{i}$ | | | |
| zi= a | | | |
| $\sigma_{\mathbf{r}}(\mathbf{r}) := \frac{a^2 \cdot \mathbf{p}_1 - b^2 \cdot \mathbf{p}_0}{b^2 - a^2} - \frac{(\mathbf{p}_1 - \mathbf{p}_0) \cdot a^2 \cdot b^2}{b^2 - a^2} \cdot \frac{1}{\mathbf{r}^2} \qquad \qquad \sigma_{\mathbf{t}}(\mathbf{r}) := \frac{a^2 \cdot \mathbf{p}_1 - b^2 \cdot \mathbf{p}_0}{b^2 - a^2} + \frac{(\mathbf{p}_1 - \mathbf{p}_0) \cdot a^2 \cdot b^2}{(b^2 - a^2) \cdot \mathbf{r}^2}$ | $u(r) \coloneqq \frac{1-\nu}{E} \cdot \frac{\left(a^2 \cdot p_1 - b^2 \cdot p_0\right) \cdot r}{b^2 - a^2} + \frac{1+\nu}{E}$ | $\frac{(\mathbf{p}_i - \mathbf{p}_o) \cdot a^2 \cdot b^2}{(b^2 - a^2) \cdot r} \qquad \sigma_{Von_Mises}(r) \coloneqq \sqrt{\sigma_t(r)^2 + \sigma_r(r)^2 - \sigma_t(r) \cdot \sigma_r(r)}$ | $\tau_{max}(r) \coloneqq 0.5 \left(\sigma_t(r) - \sigma_r(r)\right)$ |
| $\sigma_{\rm r}(r)=0{\rm psi}\qquad \qquad \sigma_{\rm t}(r)=-9072{\rm psi}$ | u(r) = -0.0003 in | $\sigma_{Von_Mises}(r) = 9072 psi$ | $\tau_{max}(r) = -4536 psi$ |
| $ \begin{array}{l} \textbf{r}:= \textbf{b} \\ \sigma_{r}(\textbf{r}):= \frac{a^{2} \cdot \textbf{p}_{1} - b^{2} \cdot \textbf{p}_{0}}{b^{2} - a^{2}} - \frac{\left(\textbf{p}_{1} - \textbf{p}_{0}\right) \cdot a^{2} \cdot b^{2}}{b^{2} - a^{2}} \cdot \frac{1}{r^{2}} \\ \sigma_{r}(\textbf{r}):= \frac{a^{2} \cdot \textbf{p}_{1} - b^{2} \cdot \textbf{p}_{0}}{b^{2} - a^{2}} + \frac{\left(\textbf{p}_{1} - \textbf{p}_{0}\right) \cdot a^{2} \cdot b^{2}}{\left(b^{2} - a^{2}\right) \cdot r^{2}} \\ \sigma_{r}(\textbf{r}):= -4493 \text{psi} \\ \end{array} $ | $u(r) := \frac{1 - v}{E} \cdot \frac{\left(a^2 \cdot p_1 - b^2 \cdot p_0\right) \cdot r}{b^2 - a^2} + \frac{1 + v}{E}$ $u(r) = -0.001 \text{ in}$ | $ \begin{array}{ll} & \underbrace{(\mathtt{p}_i - \mathtt{p}_0) \cdot a^2 \cdot b^2}_{\left(b^2 - a^2\right) \cdot r} & \sigma_{\mathrm{Von_Mises}}(r) \coloneqq \sqrt{\sigma_t(r)^2 + \sigma_r(r)^2 - \sigma_t(r) \cdot \sigma_r(r)} \\ & \sigma_{\mathrm{Von_Mises}}(r) = 4537 \mathrm{psi} \end{array} $ | $\begin{split} \tau_{\max}(r) &:= 0.5 \Big(\sigma_t(r) - \sigma_r(r) \Big) \\ \tau_{\max}(r) &= -43 \mathrm{psi} \end{split}$ |
| Stresses in the Trunnion Hub | | | |
| $a \coloneqq 9 in \qquad b \coloneqq 14 in \qquad c \coloneqq 30 in \qquad p_i \coloneqq p_{Shrink_Fit_i} \qquad p_o \coloneqq p_{Shrink_Fit_i}$ | 3 | | |
| r := a | (| | |
| $\sigma_{\mathbf{r}}(\mathbf{r}) := \frac{a^2 \cdot \mathbf{p}_{\mathbf{i}} - b^2 \cdot \mathbf{p}_{\mathbf{o}}}{b^2 - a^2} - \frac{(\mathbf{p}_{\mathbf{i}} - \mathbf{p}_{\mathbf{o}}) \cdot a^2 \cdot b^2}{b^2 - a^2} \cdot \frac{1}{\mathbf{r}^2} \qquad \qquad \sigma_{\mathbf{t}}(\mathbf{r}) := \frac{a^2 \cdot \mathbf{p}_{\mathbf{i}} - b^2 \cdot \mathbf{p}_{\mathbf{o}}}{b^2 - a^2} + \frac{(\mathbf{p}_{\mathbf{i}} - \mathbf{p}_{\mathbf{o}}) \cdot a^2 \cdot b^2}{(b^2 - a^2) \cdot r^2}$ | $u(\mathbf{r}) \coloneqq \frac{1-v}{E} \cdot \frac{\left(a^2 \cdot \mathbf{p}_1 - b^2 \cdot \mathbf{p}_0\right) \cdot \mathbf{r}}{b^2 - a^2} + \frac{1+v}{E}$ | $\frac{(\mathbf{p}_{i}-\mathbf{p}_{o})\cdot\mathbf{a}^{2}\cdot\mathbf{b}^{2}}{(\mathbf{b}^{2}-\mathbf{a}^{2})\cdot\mathbf{r}} \qquad \sigma_{Von_Mises}(\mathbf{r}) \coloneqq \sqrt{\sigma_{t}(\mathbf{r})^{2}+\sigma_{t}(\mathbf{r})\cdot\sigma_{t}(\mathbf{r})}$ | $\tau_{max}(\mathbf{r}) \coloneqq 0.5 \left(\sigma_t(\mathbf{r}) - \sigma_t(\mathbf{r})\right)$ |
| $\sigma_{\rm r}({\bf r}) = -4493.352~{ m psi}$ $\sigma_{\rm t}({\bf r}) = -13215~{ m psi}$ | u(r) = -0.0037 in | $\sigma_{\text{Von}_{\text{Mises}}}(\mathbf{r}) = 11638 \mathrm{psi}$ | $\tau_{max}(\mathbf{r}) = -4361 \mathrm{psi}$ |
| $ \begin{array}{l} \hline {\bf r}:={\bf b} \\ \sigma_z({\bf r}):=\frac{a^2\cdot {\bf p}_1-b^2\cdot {\bf p}_0}{b^2-a^2}-\frac{\left({\bf p}_1-{\bf p}_0\right)\cdot a^2\cdot b^2}{b^2-a^2}\cdot \frac{1}{{\bf r}^2} \\ \sigma_z({\bf r}):=-7052{\rm psi} \end{array} \qquad \qquad \\ \begin{array}{l} \sigma_t({\bf r}):=\frac{a^2\cdot {\bf p}_1-b^2\cdot {\bf p}_0}{b^2-a^2}+\frac{\left({\bf p}_1-{\bf p}_0\right)\cdot a^2\cdot b^2}{\left(b^2-a^2\right)\cdot {\bf r}^2} \\ \sigma_z({\bf r}):=-10656{\rm psi} \end{array} $ | $\mathbf{u}(\mathbf{r}) \coloneqq \frac{1-\nu}{E} \cdot \frac{\left(a^2 \cdot \mathbf{p}_1 - b^2 \cdot \mathbf{p}_0\right) \cdot \mathbf{r}}{b^2 - a^2} + \frac{1+\nu}{E}$ $\mathbf{u}(\mathbf{r}) = -0.0042 \text{ in}$ | $ \begin{array}{l} \underbrace{(\mathbf{p}_{1}-\mathbf{p}_{0})\cdot\mathbf{a}^{2}\cdot\mathbf{b}^{2}}_{\left(\mathbf{b}^{2}-\mathbf{a}^{2}\right)\cdot\mathbf{r}} & \sigma_{\mathrm{Von}}\mathrm{Mises}(\mathbf{r}) \coloneqq \sqrt{\sigma_{t}(\mathbf{r})^{2}+\sigma_{t}(\mathbf{r})\cdot\sigma_{t}(\mathbf{r})} \\ \sigma_{\mathrm{Von}}\mathrm{Mises}(\mathbf{r}) \coloneqq 9388 \mathrm{psi} \end{array} $ | $\begin{split} \tau_{\max}(\mathbf{r}) &:= 0.5 \big(\sigma_t(\mathbf{r}) - \sigma_x(\mathbf{r})\big) \\ \tau_{\max}(\mathbf{r}) &= -1802 \mathrm{psi} \end{split}$ |

Stresses in the Bascule Girder

| $a := 9 \text{in} \qquad b := 14 \text{in} \qquad c := 30 \text{in} \qquad p_1 := p_{\text{SI}}$ | hrink_Fit_o p _o := 0psi | | | |
|---|---|---|--|---|
| $\begin{split} \mathbf{r} &:= \mathbf{b} \\ \sigma_{\mathbf{r}}(\mathbf{r}) &:= \frac{\mathbf{b}^2 \cdot \mathbf{p}_1 - \mathbf{c}^2 \cdot \mathbf{p}_0}{\mathbf{c}^2 - \mathbf{b}^2} - \frac{(\mathbf{p}_1 - \mathbf{p}_0) \cdot \mathbf{b}^2 \cdot \mathbf{c}^2}{\mathbf{c}^2 - \mathbf{b}^2} \cdot \frac{1}{\mathbf{r}^2} \end{split}$ | $\sigma_t(\mathbf{r}) \coloneqq \frac{\mathbf{b}^2 \cdot \mathbf{p}_i - \mathbf{c}^2 \cdot \mathbf{p}_o}{\mathbf{c}^2 - \mathbf{b}^2} + \frac{\left(\mathbf{p}_i - \mathbf{p}_o\right) \cdot \mathbf{b}^2 \cdot \mathbf{c}^2}{\left(\mathbf{c}^2 - \mathbf{b}^2\right) \cdot \mathbf{r}^2}$ | $\mathbf{u}(\mathbf{r}) \coloneqq \frac{1 - v_{o}}{E_{o}} \cdot \frac{\left(\mathbf{b}^{2} \cdot \mathbf{p}_{i} - \mathbf{c}^{2} \cdot \mathbf{p}_{o}\right) \cdot \mathbf{r}}{\mathbf{c}^{2} - \mathbf{b}^{2}} + \frac{1 + v_{o}}{E_{o}} \cdot \frac{\left(\mathbf{p}_{i} - \mathbf{p}_{o}\right) \cdot \mathbf{b}^{2} \cdot \mathbf{c}^{2}}{\left(\mathbf{c}^{2} - \mathbf{b}^{2}\right) \cdot \mathbf{r}}$ | $\sigma_{\text{Von}_\text{Mises}}(r) \coloneqq \sqrt{\sigma_t(r)^2 + \sigma_z(r)^2 - \sigma_t(r) \cdot \sigma_z(r)}$ | $\tau_{\max}(\mathbf{r}) \coloneqq 0.5 \big(\sigma_t(\mathbf{r}) - \sigma_r(\mathbf{r}) \big)$ |
| $\sigma_{\rm r}({\bf r}) = -7052{\rm psi}$ | $\sigma_t(\mathbf{r}) = 10978 \text{ psi}$ | u(r) = 0.0107 in | $\sigma_{Von_Mises}(r) = 15738 psi$ | $\tau_{max}(\mathbf{r}) = 9015 \mathrm{psi}$ |
| r:= c | | | | |
| $\sigma_{\mathbf{r}}(\mathbf{r}) := \frac{\mathbf{b}^2 \cdot \mathbf{p}_1 - \mathbf{c}^2 \cdot \mathbf{p}_0}{\mathbf{c}^2 - \mathbf{b}^2} - \frac{(\mathbf{p}_1 - \mathbf{p}_0) \cdot \mathbf{b}^2 \cdot \mathbf{c}^2}{\mathbf{c}^2 - \mathbf{b}^2} \cdot \frac{1}{\mathbf{r}^2}$ | $\sigma_t(\mathbf{r}) \coloneqq \frac{\mathbf{b}^2 \cdot \mathbf{p}_i - \mathbf{c}^2 \cdot \mathbf{p}_o}{\mathbf{c}^2 - \mathbf{b}^2} + \frac{(\mathbf{p}_i - \mathbf{p}_o) \cdot \mathbf{b}^2 \cdot \mathbf{c}^2}{(\mathbf{c}^2 - \mathbf{b}^2) \cdot \mathbf{r}^2}$ | $u(\mathbf{r}) := \frac{1 - v_o}{E_o} \cdot \frac{\left(b^2 \cdot \mathbf{p}_1 - c^2 \cdot \mathbf{p}_o\right) \cdot \mathbf{r}}{c^2 - b^2} + \frac{1 + v_o}{E_o} \cdot \frac{(\mathbf{p}_1 - \mathbf{p}_o) \cdot b^2 \cdot c^2}{(c^2 - b^2) \cdot \mathbf{r}}$ | $\sigma_{Von_Mises}(\mathbf{r}) \coloneqq \sqrt{\sigma_t(\mathbf{r})^2 + \sigma_r(\mathbf{r})^2 - \sigma_t(\mathbf{r}) \cdot \sigma_r(\mathbf{r})}$ | $\tau_{max}(\mathbf{r}) \coloneqq 0.5 \big(\sigma_t(\mathbf{r}) - \sigma_r(\mathbf{r}) \big)$ |
| $\sigma_{\rm r}({\bf r}) = 0 {\rm psi}$ | $\sigma_t(\mathbf{r}) = 3927 \mathrm{psi}$ | u(r) = 0.0069 in | σ _{Von_Mises} (r) = 3927 psi | $\tau_{\max}(\mathbf{r}) = 1963 \mathrm{psi}$ |
| Shrink Fit Anaylsis of Trunnion | n, Trunnion Hub, and Outb | oard Trunnion Girder | | |
| Shrink Fit Anaylsis of Trunnic | on and Trunnion Hub | 1 | | |
| $E:= 29000000psi \ v:= 0.29 \ a:= 0.875in \ b:= 9in$ | $\mathbf{c} := 14 \mathbf{i} \mathbf{n} \qquad \boldsymbol{\delta} := 0.00478 \mathbf{i} \mathbf{n}$ | Shrink Fit Pressure PShrin | $k_{Fit_i} := \frac{E \cdot \delta}{b} \cdot \frac{(b^2 - a^2)(c^2 - b^2)}{2 \cdot b^2(c^2 - a^2)}$ PShrink_Fit_i | = 4493 psi |
| Shrink Fit Anaylsis of Trunnio | on Hub and Outboard Trun | nion Girder | × * | |
| $E := 29000000 p_{\text{si}} v := 0.29 a := 9 \text{in} \qquad b := 14 \text{in}$ | $\mathbf{a} \mathbf{c} := 21 \mathrm{in} \qquad \boldsymbol{\delta} := 0.0135 \mathrm{in}$ | Shrink Fit Pressure PShrin | $a_{k}Fit_{o} := \frac{E \cdot 5}{b} \cdot \frac{(b^2 - a^2)(c^2 - b^2)}{2 \cdot b^2(c^2 - a^2)}$ $p_{Shrink}Fit_{o}$ | = 5583 psi |
| Stresses in the Trunnion | | | | |
| $a := 0.875$ in $b := 9$ in $c := 14$ in $p_1 := 0$ | psi p _o := p _{Shrink_Fit_i} | | | |
| $\begin{aligned} \mathbf{r} &:= \mathbf{a} \\ \sigma_{\mathbf{r}}(\mathbf{r}) &:= \frac{\mathbf{a}^2 \cdot \mathbf{p}_{\mathbf{i}} - \mathbf{b}^2 \cdot \mathbf{p}_{\mathbf{o}}}{\mathbf{b}^2 - \mathbf{a}^2} - \frac{(\mathbf{p}_{\mathbf{i}} - \mathbf{p}_{\mathbf{o}}) \cdot \mathbf{a}^2 \cdot \mathbf{b}^2}{\mathbf{b}^2 - \mathbf{a}^2} \cdot \frac{1}{\mathbf{r}^2} \end{aligned}$ | $\sigma_t(\mathbf{r}) \coloneqq \frac{\mathbf{a}^2 \cdot \mathbf{p}_i - \mathbf{b}^2 \cdot \mathbf{p}_o}{\mathbf{b}^2 - \mathbf{a}^2} + \frac{(\mathbf{p}_i - \mathbf{p}_o) \cdot \mathbf{a}^2 \cdot \mathbf{b}^2}{(\mathbf{b}^2 - \mathbf{a}^2) \cdot \mathbf{r}^2}$ | $u(r) := \frac{1-v}{E} \cdot \frac{\left(\frac{a}{a} \cdot p_i - b^2 \cdot p_o\right) \cdot r}{b^2 - a^2} + \frac{1+v}{E} \cdot \frac{\left(p_i - p_o\right) \cdot a^2 \cdot b^2}{\left(b^2 - a^2\right) \cdot r}$ | $\sigma_{\text{Von}_\text{Mises}}(r) \coloneqq \sqrt{\sigma_t(r)^2 + \sigma_r(r)^2 - \sigma_t(r) \cdot \sigma_r(r)}$ | $\tau_{max}(r) \coloneqq 0.5 \Big(\sigma_t(r) - \sigma_r(r) \Big)$ |
| $\sigma_r(\mathbf{r}) = 0 \text{ psi}$ | $\sigma_t(\mathbf{r}) = -9072 \mathrm{psi}$ | u(r) = -0.0003 in | $\sigma_{\text{Von}_{\text{Mises}}(r)} = 9072 \text{psi}$ | $\tau_{max}(\mathbf{r}) = -4536\mathrm{psi}$ |
| $\begin{aligned} \mathbf{r} &:= \mathbf{b} \\ \sigma_{\mathbf{r}}(\mathbf{r}) &:= \frac{\mathbf{a}^2 \cdot \mathbf{p}_1 - \mathbf{b}^2 \cdot \mathbf{p}_0}{\mathbf{b}^2 - \mathbf{a}^2} - \frac{(\mathbf{p}_1 - \mathbf{p}_0) \cdot \mathbf{a}^2 \cdot \mathbf{b}^2}{\mathbf{b}^2 - \mathbf{a}^2} \cdot \frac{1}{\mathbf{r}^2} \\ \sigma_{\mathbf{r}}(\mathbf{r}) &= -4493 \text{psi} \end{aligned}$ | $\begin{split} \sigma_t(\mathbf{r}) &:= \frac{a^2 \cdot \mathbf{p}_1 - b^2 \cdot \mathbf{p}_0}{b^2 - a^2} + \frac{(\mathbf{p}_1 - \mathbf{p}_0) \cdot a^2 \cdot b^2}{(b^2 - a^2) \cdot r^2} \\ \sigma_t(\mathbf{r}) &= -4579 \text{psi} \end{split}$ | $\begin{split} u(r) &:= \frac{1 - v}{E} \cdot \frac{\left(a^2 \cdot p_1 - b^2 \cdot p_0\right) \cdot r}{b^2 - a^2} + \frac{1 + v}{E} \cdot \frac{\left(p_1 - p_0\right) \cdot a^2 \cdot b^2}{\left(b^2 - a^2\right) \cdot r} \\ u(r) &= -0.001 \text{ in} \end{split}$ | $\begin{split} \sigma_{\mathrm{Von}_\mathrm{Mises}}(\mathbf{r}) &\coloneqq \sqrt{\sigma_t(\mathbf{r})^2 + \sigma_t(\mathbf{r})^2 - \sigma_t(\mathbf{r}) \cdot \sigma_t(\mathbf{r})} \\ \sigma_{\mathrm{Von}_\mathrm{Mises}}(\mathbf{r}) &= 4537 \mathrm{psi} \end{split}$ | $\begin{split} \tau_{\max}(r) &:= 0.5 \big(\sigma_t(r) - \sigma_t(r) \big) \\ \tau_{\max}(r) &= -43 \mathrm{psi} \end{split}$ |

Stresses in the Trunnion Hub

r := a

 $a:=9in \qquad b:=14in \qquad c:=21in \qquad p_i:=p_{Shrink_Fit_i} \qquad p_o:=p_{Shrink_Fit_o}$

$$\sigma_{r}(r) := \frac{a^{2} \cdot p_{i} - b^{2} \cdot p_{o}}{b^{2} - a^{2}} - \frac{(p_{i} - p_{o}) \cdot a^{2} \cdot b^{2}}{b^{2} - a^{2}} \frac{1}{r^{2}}$$

$$\sigma_{t}(r) := \frac{a^{2} \cdot p_{i} - b^{2} \cdot p_{o}}{b^{2} - a^{2}} + \frac{(p_{i} - p_{o}) \cdot a^{2} \cdot b^{2}}{(b^{2} - a^{2}) \cdot r^{2}}$$

$$u(r) := \frac{1 - v}{E} \cdot \frac{(a^{2} \cdot p_{i} - b^{2} \cdot p_{o}) \cdot r}{b^{2} - a^{2}} + \frac{1 + v}{E} \cdot \frac{(p_{i} - p_{o}) \cdot a^{2} \cdot b^{2}}{(b^{2} - a^{2}) \cdot r}$$

$$\sigma_{Von_{i}Mises}(r) := \sqrt{\sigma_{t}(r)^{2} + \sigma_{r}(r)^{2} - \sigma_{t}(r) \cdot \sigma_{r}(r)}$$

$$\tau_{max}(r) := 0.5(\sigma_{t}(r) - \sigma_{r}(r))$$

$$r := b$$

$$\sigma_{z}(r) := \frac{a^{2} \cdot p_{1} - b^{2} \cdot p_{0}}{b^{2} - a^{2}} - \frac{(p_{1} - p_{0}) \cdot a^{2} \cdot b^{2}}{b^{2} - a^{2}} \cdot \frac{1}{r^{2}} \qquad \sigma_{t}(r) := \frac{a^{2} \cdot p_{1} - b^{2} \cdot p_{0}}{b^{2} - a^{2}} + \frac{(p_{1} - p_{0}) \cdot a^{2} \cdot b^{2}}{(b^{2} - a^{2}) \cdot r^{2}} \qquad u(r) := \frac{1 - v}{E} \cdot \frac{(a^{2} \cdot p_{1} - b^{2} \cdot p_{0}) \cdot r}{b^{2} - a^{2}} + \frac{1 + v}{E} \cdot \frac{(p_{1} - p_{0}) \cdot a^{2} \cdot b^{2}}{(b^{2} - a^{2}) \cdot r} \qquad \sigma_{Von_Mises}(r) := \sqrt{\sigma_{t}(r)^{2} + \sigma_{t}(r)^{2} - \sigma_{t}(r) \cdot \sigma_{t}(r)} \qquad \tau_{max}(r) := 0.5(\sigma_{t}(r) - \sigma_{t}(r))$$

Stresses in the Outboard Trunnion Girder

b := 14in c := 21in a := 9in pi := pShrink_Fit_o p_o := 0psi r := b $\sigma_{\mathbf{r}}(\mathbf{r}) := \frac{\mathbf{b}^2 \cdot \mathbf{p}_{\mathbf{i}} - \mathbf{c}^2 \cdot \mathbf{p}_{\mathbf{o}}}{\mathbf{c}^2 - \mathbf{b}^2} - \frac{\left(\mathbf{p}_{\mathbf{i}} - \mathbf{p}_{\mathbf{o}}\right) \cdot \mathbf{b}^2 \cdot \mathbf{c}^2}{\mathbf{c}^2 - \mathbf{b}^2} \cdot \frac{1}{\mathbf{r}^2} \qquad \qquad \sigma_{\mathbf{t}}(\mathbf{r}) := \frac{\mathbf{b}^2 \cdot \mathbf{p}_{\mathbf{i}} - \mathbf{c}^2 \cdot \mathbf{p}_{\mathbf{o}}}{\left(\mathbf{c}^2 - \mathbf{b}^2\right) \cdot \mathbf{r}^2} \qquad \qquad \mathbf{u}(\mathbf{r}) := \frac{1 - v_{\mathbf{o}}}{\mathbf{E}_{\mathbf{o}}} \cdot \frac{\left(\mathbf{b}^2 \cdot \mathbf{p}_{\mathbf{i}} - \mathbf{c}^2 \cdot \mathbf{p}_{\mathbf{o}}\right) \cdot \mathbf{r}}{\mathbf{c}^2 - \mathbf{b}^2} + \frac{1 + v_{\mathbf{o}}}{(\mathbf{c}^2 - \mathbf{b}^2) \cdot \mathbf{r}^2} - \frac{(\mathbf{p}_{\mathbf{i}} - \mathbf{p}_{\mathbf{o}}) \cdot \mathbf{b}^2 \cdot \mathbf{c}^2}{(\mathbf{c}^2 - \mathbf{b}^2) \cdot \mathbf{r}^2} = \mathbf{u}(\mathbf{r}) := \frac{1 - v_{\mathbf{o}}}{\mathbf{E}_{\mathbf{o}}} \cdot \frac{\left(\mathbf{b}^2 \cdot \mathbf{p}_{\mathbf{i}} - \mathbf{c}^2 \cdot \mathbf{p}_{\mathbf{o}}\right) \cdot \mathbf{r}}{\mathbf{c}^2 - \mathbf{b}^2} + \frac{1 + v_{\mathbf{o}}}{(\mathbf{c}^2 - \mathbf{b}^2) \cdot \mathbf{r}^2} = \mathbf{u}(\mathbf{r}) := \frac{1 - v_{\mathbf{o}}}{\mathbf{E}_{\mathbf{o}}} \cdot \frac{\left(\mathbf{b}^2 \cdot \mathbf{p}_{\mathbf{i}} - \mathbf{c}^2 \cdot \mathbf{p}_{\mathbf{o}}\right) \cdot \mathbf{r}}{(\mathbf{c}^2 - \mathbf{b}^2) \cdot \mathbf{r}^2}$ $\sigma_{Von_Mises}(r) \coloneqq \sqrt{\sigma_t(r)^2 + \sigma_r(r)^2 - \sigma_t(r) \cdot \sigma_r(r)}$ $\tau_{max}(\mathbf{r}) := 0.5 (\sigma_t(\mathbf{r}) - \sigma_r(\mathbf{r}))$ $\sigma_r(\mathbf{r}) = -5583 \, \text{psi}$ σ_t(r) = 14516 psi u(r) = 0.0133 in $\tau_{\max}(\mathbf{r}) = 10050 \, \mathrm{psi}$ σ_{Von_Mises}(r) = 17970 psi r := c $\sigma_{\mathbf{i}}(\mathbf{r}) := \frac{\mathbf{b}^2 \cdot \mathbf{p}_{\mathbf{i}} - \mathbf{c}^2 \cdot \mathbf{p}_{\mathbf{o}}}{\mathbf{c}^2 - \mathbf{b}^2} - \frac{\left(\mathbf{p}_{\mathbf{i}} - \mathbf{p}_{\mathbf{o}}\right) \cdot \mathbf{b}^2 \cdot \mathbf{c}^2}{\mathbf{c}^2 - \mathbf{b}^2} + \frac{\mathbf{b}^2 \cdot \mathbf{p}_{\mathbf{i}} - \mathbf{c}^2 \cdot \mathbf{p}_{\mathbf{o}}}{\mathbf{c}^2 - \mathbf{b}^2} + \frac{\left(\mathbf{p}_{\mathbf{i}} - \mathbf{p}_{\mathbf{o}}\right) \cdot \mathbf{b}^2 \cdot \mathbf{c}^2}{\left(\mathbf{c}^2 - \mathbf{b}^2\right) \cdot \mathbf{r}^2} \qquad \mathbf{u}(\mathbf{r}) := \frac{1 - \mathbf{v}_{\mathbf{o}}}{\mathbf{E}_{\mathbf{o}}} \cdot \frac{\left(\mathbf{b}^2 \cdot \mathbf{p}_{\mathbf{i}} - \mathbf{c}^2 \cdot \mathbf{p}_{\mathbf{o}}\right) \cdot \mathbf{r}}{\mathbf{c}^2 - \mathbf{b}^2} + \frac{1 + \mathbf{v}_{\mathbf{o}}}{\mathbf{E}_{\mathbf{o}}} \cdot \frac{\left(\mathbf{p}_{\mathbf{i}} - \mathbf{p}_{\mathbf{o}}\right) \cdot \mathbf{b}^2 \cdot \mathbf{c}^2}{\mathbf{c}^2 - \mathbf{b}^2} = \mathbf{u}(\mathbf{r})$ $\sigma_{Von_Mises}(\mathbf{r}) \coloneqq \sqrt{\sigma_t(\mathbf{r})^2 + \sigma_r(\mathbf{r})^2 - \sigma_t(\mathbf{r}) \cdot \sigma_r(\mathbf{r})}$ $\tau_{max}(\mathbf{r}) := 0.5 (\sigma_t(\mathbf{r}) - \sigma_r(\mathbf{r}))$ $\sigma_{\rm r}({\bf r}) = 0 \, {\rm psi}$ $\sigma_t(\mathbf{r}) = 8933 \, \mathrm{psi}$ u(r) = 0.011 in $\sigma_{Von Mises}(r) = 8933 \, psi$ $\tau_{max}(\mathbf{r}) = 4467 \, \mathrm{psi}$

Shrink Fit Anaylsis of Trunnion, Diaphragm, and Center Trunnion Girder

| Shrink Fit Anaylsis of Trunn | ion and Diaphragm | 1 | | |
|--|---|---|---|--|
| $E:= 29000000psi \ v:= 0.29 \ a:= 0.875 in \ b:= 4$ | $5in c := 11in \qquad \delta := 0.0057in$ | Shrink Fit Pressure p | $P_{\text{Shrink}_{i}} = \frac{E \cdot \delta}{b} \cdot \frac{(b^2 - a^2)(c^2 - b^2)}{c \cdot c^2 - b^2}$ PShrink_Fi | t_i = 14808 psi |
| Shrink Fit Anaylsis of Diaph | ragm and Center Trunnion G | irder | 2-b (c – a) | |
| $E := 29000000 psi \ \nu := 0.29 \ a := 4.5 in \ b := 1$ | lin e := 21in õ := 0.0115in | Shrink Fit Pressure p | $P_{Shrink_Fit_0} := \frac{E \cdot \delta}{b} \cdot \frac{(b^2 - a^2)(c^2 - b^2)}{2 \cdot b^2(c^2 - a^2)} \qquad P_{Shrink_Fit_0}$ | _{t_0} = 9600 psi |
| Stresses in the Trunnion | | | 2.0 (c a) | |
| $a:=0.875 {\rm in} b:=4.5 {\rm in} c:=11 {\rm in} \qquad p_1:=0.875 {\rm in} c:=11 {\rm in} \ c:=11 {\rm i$ | 0psi p _o := p _{Shrink_Fit_i} | | | |
| $\begin{split} \mathbf{r} &:= \mathbf{a} \\ \sigma_{\mathbf{r}}(\mathbf{r}) &:= \frac{\mathbf{a}^2 \cdot \mathbf{p}_{\mathbf{i}} - \mathbf{b}^2 \cdot \mathbf{p}_{\mathbf{o}}}{\mathbf{b}^2 - \mathbf{a}^2} - \frac{(\mathbf{p}_{\mathbf{i}} - \mathbf{p}_{\mathbf{o}}) \cdot \mathbf{a}^2 \cdot \mathbf{b}^2}{\mathbf{b}^2 - \mathbf{a}^2} \cdot \frac{1}{\mathbf{r}^2} \end{split}$ | $\sigma_t(\mathbf{r}) \coloneqq \frac{\mathbf{a}^2 \cdot \mathbf{p}_1 - \mathbf{b}^2 \cdot \mathbf{p}_o}{\mathbf{b}^2 - \mathbf{a}^2} + \frac{(\mathbf{p}_1 - \mathbf{p}_o) \cdot \mathbf{a}^2 \cdot \mathbf{b}^2}{(\mathbf{b}^2 - \mathbf{a}^2) \cdot \mathbf{r}^2}$ | $u(r) := \frac{1-\nu}{E} \cdot \frac{\left(\frac{a}{a} \cdot p_{1} - b^{2} \cdot p_{0}\right) \cdot r}{b^{2} - a^{2}} + \frac{1+\nu}{E} \cdot \frac{\left(p_{1} - p_{0}\right) \cdot a^{2} \cdot b^{2}}{\left(b^{2} - a^{2}\right) \cdot r}$ | $\sigma_{\text{Von_Mises}}(r) := \sqrt{\sigma_t(r)^2 + \sigma_t(r)^2 - \sigma_t(r) \cdot \sigma_t(r)}$ | $\tau_{max}(r) \coloneqq 0.5 \big(\sigma_t(r) - \sigma_t(r)\big)$ |
| $\sigma_{r}(r) = 0 \text{ psi}$ | $\sigma_t(\mathbf{r}) = -30781 \mathrm{psi}$ | u(r) = -0.0009 in | $\sigma_{\text{Von}_{\text{Mises}}}(\mathbf{r}) = 30781 \text{psi}$ | $\tau_{\max}(\mathbf{r}) = -15390 \mathrm{psi}$ |
| $\begin{split} & \mathbf{r} := \mathbf{b} \\ & \sigma_1(\mathbf{r}) := \frac{\mathbf{a}^2 \cdot \mathbf{p}_1 - \mathbf{b}^2 \cdot \mathbf{p}_0}{\mathbf{b}^2 - \mathbf{a}^2} - \frac{(\mathbf{p}_1 - \mathbf{p}_0) \cdot \mathbf{a}^2 \cdot \mathbf{b}^2}{\mathbf{b}^2 - \mathbf{a}^2} \cdot \frac{1}{\mathbf{r}^2} \\ & \sigma_1(\mathbf{r}) = -14808 \mathbf{p}_0 \mathbf{i} \end{split}$ | $\begin{split} \sigma_t(r) &:= \frac{a^2 \cdot p_1 - b^2 \cdot p_0}{b^2 - a^2} + \frac{(p_1 - p_0) \cdot a^2 \cdot b^2}{(b^2 - a^2) \cdot r^2} \\ \sigma_t(r) &= -15972 psi \end{split}$ | $\begin{split} u(t) &:= \frac{1-\nu}{E} \cdot \frac{\left(a^2 \cdot p_1 - b^2 \cdot p_0\right) \cdot r}{b^2 - a^2} + \frac{1+\nu}{E} \cdot \frac{\left(p_1 - p_0\right) \cdot a^2 \cdot b^2}{\left(b^2 - a^2\right) \cdot r} \\ u(t) &= -0.0018 \text{ in} \end{split}$ | $\sigma_{\text{Von}_Mises}(r) \coloneqq \sqrt{\sigma_t(r)^2 + \sigma_r(r)^2 - \sigma_t(r) \cdot \sigma_r(r)}$ $\sigma_{\text{Von}_Mises}(r) = 15423 \text{ psi}$ | $\begin{split} \tau_{\max}(r) &:= 0.5 \big(\sigma_r(r) - \sigma_r(r)\big) \\ \tau_{\max}(r) &= -582 \text{psi} \end{split}$ |
| Stresses in the Diaphragm | | | | |
| $a:=4.5in b:=11in c:=21in \qquad p_1:=r:=a$ | p _{Shrink_Fit_i} p _o := p _{Shrink_Fit_o} | | | |
| $\sigma_{1}(r):=\frac{a^{2}\cdot p_{1}-b^{2}\cdot p_{0}}{b^{2}-a^{2}}-\frac{(p_{1}-p_{0})\cdot a^{2}\cdot b^{2}}{b^{2}-a^{2}}\cdot \frac{1}{r^{2}}$ | $\sigma_t(\mathbf{r}) \coloneqq \frac{\mathbf{a}^2 \cdot \mathbf{p}_i - \mathbf{b}^2 \cdot \mathbf{p}_o}{\mathbf{b}^2 - \mathbf{a}^2} + \frac{(\mathbf{p}_i - \mathbf{p}_o) \cdot \mathbf{a}^2 \cdot \mathbf{b}^2}{(\mathbf{b}^2 - \mathbf{a}^2) \cdot \mathbf{r}^2}$ | $u(r) := \frac{1 - v}{E} \cdot \frac{\left(a^2 \cdot p_1 - b^2 \cdot p_0\right) \cdot r}{b^2 - a^2} + \frac{1 + v}{E} \cdot \frac{\left(p_1 - p_0\right) \cdot a^2 \cdot b^2}{\left(b^2 - a^2\right) \cdot r}$ | $\sigma_{\text{Von}_{\text{Mises}}}(r) \coloneqq \sqrt{\sigma_{t}(r)^{2} + \sigma_{t}(r)^{2} - \sigma_{t}(r) \cdot \sigma_{t}(r)}$ | $\tau_{\max}(\mathbf{r}) \coloneqq 0.5 \big(\sigma_t(\mathbf{r}) - \sigma_t(\mathbf{r}) \big)$ |
| $\sigma_{1}(r) = -14808.402 \text{ psi}$ | $\sigma_t(\mathbf{r}) = -2297 \mathrm{psi}$ | u(r) = 0.0003 in | $\sigma_{Von_Mises}(r) = 13804 psi$ | $\tau_{\max}(\mathbf{r}) = 6256\mathrm{psi}$ |
| $\mathbf{r} := \mathbf{b}$ $\sigma_{1}(\mathbf{r}) := \frac{\mathbf{a}^{2} \cdot \mathbf{p}_{1} - \mathbf{b}^{2} \cdot \mathbf{p}_{0}}{\mathbf{b}^{2} - \mathbf{a}^{2}} - \frac{(\mathbf{p}_{1} - \mathbf{p}_{0}) \cdot \mathbf{a}^{2} \cdot \mathbf{b}^{2}}{\mathbf{b}^{2} - \mathbf{a}^{2}} \cdot \frac{1}{\mathbf{r}^{2}}$ | $\sigma_t(r) := \frac{a^2 \cdot p_i - b^2 \cdot p_o}{b^2 - a^2} + \frac{(p_i - p_o) \cdot a^2 \cdot b^2}{(b^2 - a^2) \cdot r^2}$ | $u(r) := \frac{1 - v}{E} \cdot \frac{\left(a^2 \cdot p_1 - b^2 \cdot p_0\right) \cdot r}{b^2 - a^2} + \frac{1 + v}{E} \cdot \frac{\left(p_1 - p_0\right) \cdot a^2 \cdot b^2}{\left(b^2 - a^2\right) \cdot r}$ | $\sigma_{\text{Von}_{\text{Mises}}}(\mathbf{r}) \coloneqq \sqrt{\sigma_{t}(\mathbf{r})^{2} + \sigma_{t}(\mathbf{r})^{2} - \sigma_{t}(\mathbf{r}) \cdot \sigma_{t}(\mathbf{r})}$ | $\tau_{\max}(\mathbf{r}) \coloneqq 0.5 \big(\sigma_{\mathbf{t}}(\mathbf{r}) - \sigma_{\mathbf{r}}(\mathbf{r}) \big)$ |
| $\sigma_{\rm r}({\rm r}) = -9600{\rm psi}$ | $\sigma_t(\mathbf{r}) = -7506 \mathrm{psi}$ | u(r) = -0.0018 in | $\sigma_{\text{Von}_{\text{Mises}}}(r) = 8743 \text{psi}$ | $\tau_{max}(r) = 1047 psi$ |

Stresses in the Center Trunnion Girder

a := 4.5in b := 11in c := 21in p_i := p_{Shrink_Fit_o} p_o := 0psi r := b $\sigma_{\mathbf{r}}(\mathbf{r}) := \frac{\mathbf{b}^{2} \cdot \mathbf{p}_{\mathbf{i}} - \mathbf{c}^{2} \cdot \mathbf{p}_{\mathbf{o}}}{\mathbf{c}^{2} - \mathbf{b}^{2}} - \frac{(\mathbf{p}_{\mathbf{i}} - \mathbf{p}_{\mathbf{o}}) \cdot \mathbf{b}^{2} \cdot \mathbf{c}^{2}}{\mathbf{c}^{2} - \mathbf{b}^{2}} \cdot \frac{1}{\mathbf{r}^{2}} \qquad \sigma_{\mathbf{t}}(\mathbf{r}) := \frac{\mathbf{b}^{2} \cdot \mathbf{p}_{\mathbf{i}} - \mathbf{c}^{2} \cdot \mathbf{p}_{\mathbf{o}}}{\mathbf{c}^{2} - \mathbf{b}^{2}} + \frac{(\mathbf{p}_{\mathbf{i}} - \mathbf{p}_{\mathbf{o}}) \cdot \mathbf{b}^{2} \cdot \mathbf{c}^{2}}{(\mathbf{c}^{2} - \mathbf{b}^{2}) \cdot \mathbf{r}^{2}} \qquad u(\mathbf{r}) := \frac{1 - v_{\mathbf{o}}}{\mathbf{c}} \cdot \frac{(\mathbf{b}^{2} \cdot \mathbf{p}_{\mathbf{i}} - \mathbf{c}^{2} \cdot \mathbf{p}_{\mathbf{o}}) \cdot \mathbf{r}}{\mathbf{c}^{2} - \mathbf{b}^{2}} + \frac{1 + v_{\mathbf{o}}}{(\mathbf{c}^{2} - \mathbf{b}^{2}) \cdot \mathbf{r}^{2}} \qquad \sigma_{\mathrm{Von}_\mathrm{Mises}}(\mathbf{r}) := \sqrt{\sigma_{\mathbf{t}}(\mathbf{r})^{2} + \sigma_{\mathbf{t}}(\mathbf{r})^{2} - \sigma_{\mathbf{t}}(\mathbf{r}) \cdot \sigma_{\mathbf{r}}(\mathbf{r})} \qquad \tau_{\mathrm{max}}(\mathbf{r}) := 0.5(\sigma_{\mathbf{t}}(\mathbf{r}) - \sigma_{\mathbf{t}}(\mathbf{r})) \cdot \mathbf{r}^{2} \cdot \mathbf{r}^{2}$ $\sigma_{Von_Mises}(r) = 23200 \, psi$ $\tau_{max}(\mathbf{r}) = 13230 \, \mathrm{psi}$ $\sigma_{\rm r}({\rm r}) = -9600 \, {\rm psi}$ $\sigma_{*}(r) = 16860 \text{ psi}$ u(r) = 0.0127 in $\mathbf{r} := \mathbf{c}$ $\sigma_{\mathbf{r}}(\mathbf{r}) := \frac{\mathbf{b}^{2} \cdot \mathbf{p}_{\mathbf{i}} - \mathbf{c}^{2} \cdot \mathbf{p}_{\mathbf{o}}}{\mathbf{c}^{2} - \mathbf{b}^{2}} - \frac{\left(\mathbf{p}_{\mathbf{i}} - \mathbf{p}_{\mathbf{o}}\right) \cdot \mathbf{b}^{2} \cdot \mathbf{c}^{2}}{\mathbf{c}^{2} - \mathbf{b}^{2}} + \frac{\left(\mathbf{p}_{\mathbf{i}} - \mathbf{p}_{\mathbf{o}}\right) \cdot \mathbf{b}^{2} \cdot \mathbf{c}^{2}}{(\mathbf{c}^{2} - \mathbf{b}^{2}) \cdot \mathbf{r}^{2}} \\ \mathbf{u}(\mathbf{r}) := \frac{1 - v_{\mathbf{o}}}{E_{\mathbf{o}}} \cdot \frac{\left(\mathbf{b}^{2} \cdot \mathbf{p}_{\mathbf{i}} - \mathbf{c}^{2} \cdot \mathbf{p}_{\mathbf{o}}\right) \cdot \mathbf{r}}{\mathbf{c}^{2} - \mathbf{b}^{2}} + \frac{1 + v_{\mathbf{o}}}{(\mathbf{c}^{2} - \mathbf{b}^{2}) \cdot \mathbf{r}^{2}} \\ \sigma_{\mathrm{Von}_\mathrm{Mises}}(\mathbf{r}) := \sqrt{\sigma_{\mathbf{t}}(\mathbf{r})^{2} + \sigma_{\mathbf{r}}(\mathbf{r})^{2} - \sigma_{\mathbf{t}}(\mathbf{r}) \cdot \sigma_{\mathbf{r}}(\mathbf{r})} \\ \sigma_{\mathrm{Von}_\mathrm{Mises}}(\mathbf{r}) := \sqrt{\sigma_{\mathbf{t}}(\mathbf{r})^{2} + \sigma_{\mathbf{r}}(\mathbf{r})^{2} - \sigma_{\mathbf{t}}(\mathbf{r}) \cdot \sigma_{\mathbf{r}}(\mathbf{r})} \\ \sigma_{\mathrm{Von}_\mathrm{Mises}}(\mathbf{r}) := \sqrt{\sigma_{\mathbf{t}}(\mathbf{r})^{2} + \sigma_{\mathbf{r}}(\mathbf{r})^{2} - \sigma_{\mathbf{t}}(\mathbf{r}) \cdot \sigma_{\mathbf{r}}(\mathbf{r})} \\ \sigma_{\mathrm{Von}_\mathrm{Mises}}(\mathbf{r}) := \sqrt{\sigma_{\mathbf{t}}(\mathbf{r})^{2} + \sigma_{\mathbf{r}}(\mathbf{r})^{2} - \sigma_{\mathbf{t}}(\mathbf{r}) \cdot \sigma_{\mathbf{r}}(\mathbf{r})} \\ \sigma_{\mathrm{Mises}}(\mathbf{r}) := \sqrt{\sigma_{\mathbf{t}}(\mathbf{r})^{2} + \sigma_{\mathbf{r}}(\mathbf{r})^{2} - \sigma_{\mathbf{t}}(\mathbf{r}) \cdot \sigma_{\mathbf{r}}(\mathbf{r})} \\ \sigma_{\mathrm{Mises}}(\mathbf{r}) := \sqrt{\sigma_{\mathbf{t}}(\mathbf{r})^{2} + \sigma_{\mathbf{r}}(\mathbf{r})^{2} - \sigma_{\mathbf{t}}(\mathbf{r}) \cdot \sigma_{\mathbf{r}}(\mathbf{r})} \\ \sigma_{\mathrm{Mises}}(\mathbf{r}) := \sqrt{\sigma_{\mathbf{t}}(\mathbf{r})^{2} + \sigma_{\mathbf{r}}(\mathbf{r})^{2} - \sigma_{\mathbf{t}}(\mathbf{r}) \cdot \sigma_{\mathbf{r}}(\mathbf{r})} \\ \sigma_{\mathrm{Mises}}(\mathbf{r}) := \sqrt{\sigma_{\mathbf{t}}(\mathbf{r})^{2} + \sigma_{\mathbf{r}}(\mathbf{r})^{2} - \sigma_{\mathbf{t}}(\mathbf{r}) \cdot \sigma_{\mathbf{r}}(\mathbf{r})} \\ \sigma_{\mathrm{Mises}}(\mathbf{r}) := \sqrt{\sigma_{\mathbf{t}}(\mathbf{r})^{2} + \sigma_{\mathbf{t}}(\mathbf{r})^{2} - \sigma_{\mathbf{t}}(\mathbf{r}) \cdot \sigma_{\mathbf{r}}(\mathbf{r})} \\ \sigma_{\mathrm{Mises}}(\mathbf{r}) := \sqrt{\sigma_{\mathbf{t}}(\mathbf{r})^{2} + \sigma_{\mathbf{t}}(\mathbf{r})^{2} - \sigma_{\mathbf{t}}(\mathbf{r})^{2} + \sigma_{\mathbf{t}}(\mathbf{r})^{2} - \sigma_{\mathbf{t}}(\mathbf{r})^{2} - \sigma_{\mathbf{t}}(\mathbf{r})^{2} + \sigma_{\mathbf{t}}(\mathbf{r})^{2} - \sigma_{\mathbf{t}}(\mathbf{r})^{2} + \sigma_{\mathbf{t}}(\mathbf{r}$ $\tau_{max}(\mathbf{r}) := 0.5 (\sigma_t(\mathbf{r}) - \sigma_r(\mathbf{r}))$ $\sigma_{\rm r}({\rm r}) = -0 \, {\rm psi}$ $\sigma_{Von_Mises}(r) = 7260 \, psi$ $\sigma_t(\mathbf{r}) = 7260 \, \mathrm{psi}$ u(r) = 0.009 in $\tau_{max}(\mathbf{r}) = 3630 \, \mathrm{psi}$